Arithmetical Trigonometry:

BEING THE

SOLUTION

Of all the Usual

CASES

IN

Plain Trigonometry

BY COMMON

ARITHMETICK,

Without any TABLES whatsoever.

To which is added

An Easy, Exact and Speedy Method for making the TABLES of Natural Sines, Tangents and Secants: As also the making of the TABLES of Logarithms, and of the Artificial Sines, Tangents and Secants. With some useful TABLES in GUNNERY.

BY *MARK FORSTER*.

London; Printed for *Richard Mount*, at the *Postern* on *Tower-Hill*, 1700. Where you may have all sorts of Mathematical and Sea-Books.

To the Candid Reader by way of Introduction and Preface.

HAving Observed that the Doctrine of Trigonometry depends wholly upon the help of Tables, and Considering how many accidents and impediments may intervene and hinder that conveniency. Many have set about the performance of plain Trigonometry by Natural Arithmetick, and most upon this ground, viz. That having assigned the Hypothenuse of a Right Angled plain Triangle to be alwayes 1,00, they found the base and perpendicular thereto, or (which is all one) the Sines and Cosines in Decimal Numbers to every point half, and (sometimes) quarter point of the Angle at the base to 45 d. which Numbers were reserved as fixt or stationary Numbers whereby (as in similar Triangles) to find the parts unknown in the Triangle proposed. Now although this method (remembring all those numbers) was not to be despised, yet to perform plain Trigonometry hereby to any degree or minute was impossible without Tables of those Numbers, which would have been the same as the Table

Natural Sines; and impossible to be committed to memory. Therefore to remedy this inconveniency in order to the more exact performance of plain Trigonometry Arithmetically with most ease to the memory, there has lately been invented a general Rule to solve a right Angled plain Triangle by Arithmetick, to 3 or 4 places of figures, and now for common good the same is made publick, with a method shewing (to any reasonable Capacity) how the whole Doctrine of plain Trigonometry may be accommodated thereto, and thereby exactly calculated to three or four places of figures, which is exact enough for any common use, and how far it may be serviceable as well at Land as Sea, the impartial Practitioner may judge, it has been long desired by many, but never (that I know of) attained unto or published till now. And it holds as well when the Angles are given in any degrees and minutes whatsoever, as when in even degrees, or upon even points halves or quarter points of the Compass, being no further burdensome to the Memory then the remembrance of a few lines, which is inconsiderable in a Rule of such universal use. Now this general Rule or Fundamen-

al

al Axiom, ought in the first place to be well imprinted in the memory before you procede to the Resolution of plain Triangles thereby. And now having premised thus much by way of Introduction. I shall in the ensuing part of this discourse give the Reader a Breviat of the Subject of each part contained in this Miscellaneous Manual, viz. 1. *The Doctrine of right Angled plain Triangles, all the Cases ~~therein with~~ examples of all varieties that can happen, fairly resolved Arithmetically,* to page 19.

2. *The Doctrine of Oblique plain Triangles compleatly handled in all the Cases, with Examples of all the varieties that each Case admits of, all resolved by Natural Arithmetick,* to pag. 63.

3. *To the end that the Courteous Reader may be compleatly furnished with whatever is subservient in this Arithmetical Trigonometry. I have added a full Treatise of* Decimal Arithmetick, to pag. 86. *and there begun a discourse of good use concerning Multiplicators and Divisors,* viz. *How to Convert a given divisor into a Multiplicator, that shall effect the same as the divisor given and the contrary, and this is explain-*

ed, to page 89. *with some uses of this Conversion in fixt Proportions,* to pag. 94. *as also* to p. 99. *by some Questions in the rule of Three, but more amply the advantage of the said Conversion appears in several Questions in rules of plural proportion, such as both kinds of Fellowship, and Alligation each sort, and this,* to page 122.

4. *The Extraction of the Square-root by very plain and easy rules, with several Examples both in Integers and Fractions, for better instruction of the Learner, this being much used in Arithmetical* Trigonometry, unto page 141.

5. *A very plain exact and easie method how to make the Tables of Natural Sines, Tangents and Secants to any Radius, or to Examine the certainty of any Tables already calculated, with such Explications as renders the subject thereof very intelligible,* to page 170.

6. *The Construction or making the Tables of Logarithms, by the best and easiest method hitherto found out or published, in the Construction or Examination of which Tables the advantage of converting Divisors* in-

into Multiplicators, and the contrary is farther manifested, as far as page 201.

7. *The making of the Artificial, Sines, Tangents and Secants, by brief but plain and intelligible Directions.*

8. *And lastly, The manner of calculating those useful Tables of Randoms or Ranges in Gunnery, by some called tables of Horizontal distances, by an easie proportion which hath heretofore been published by some and approved by others of good knowledge and esteem.*

All which is performed with as much perspicuity and plainness, as the subject would admit, but if some things seem too prolix to the more Learned sort. I hope the fault (if any) is pardonable, for that the matter herein contained was chiefly intended for Learners and not for the Learned: although I doubt not but those that have made a considerable proficiency in the Mathematicks may herein find some things altogether new, and others probably which may be both new to them and of good use. Thus having given a brief account to my impartial Reader both of the whole and its parts, hoping few Errors have escaped that

are

are material, for that there hath been great care used to prevent the same; yet possibly some may have been overlooked, which I hope the Readers Candour will prompt him to amend. For as it is impossible to find a man faultless, So is it likewise as difficult to find a book without Errours. To this I may further add that these inferiour things meeting with good Reception may in due time incourage the publication of other things more abstruse and sublime; which I doubt not but will be very acceptable as well as useful, to those Mathematicians of a higher sphere, which as they are more knowing and quick sighted, so a particular prerogative of honour and respect is due unto them, and care shall be taken to present the next in a better method and order; and in the interim I remain, according to my power, a true Friend and well Wisher to the Publick-Weal.

M. Forster.

Arithmetical Trigonometry,

OR,

The Doctrine of Right Angled Plain Triangles, performed by Natural Arithmetick.

BEfore we proceed to the Calculation of Right Angled plain Triangles by Natural Arithmetick, I shall premise as chiefly preparatory thereunto.

That having the Angles in any Right angled plain Triangle, and assuming the Leg opposite to the lesser angle to be 1,00, &c. This following Rule, will universally give the Hypothenuse and other Leg.

The Rule is,

DIvide 172 by the Angle opposite to the lesser Leg, (which Leg must always be assumed 1,00 &c.) *Square the quotient, from which abate 3, and Extract the Square Root of the Remainder. This Root substracted from twice the Quotient, one third part of the Remainder is the Hypothenuse; and the Hypothenuse Doubled and Substracted from the said Quotient leaves the other Leg.*

Therefore when any Right angled plain Triangle is propoſed for Solution by this method, having the angles given, you ~~muſt firſt always aſſume another~~ right angled plain Triangle ~~having the ſame angles with that propoſed~~, and the Leg oppoſite to the leſſer angle to be 1,00 &c. whereby find the Hypothenuſe and other Leg in that aſſumed Triangle by the foregoing Rule, which done, thoſe two Triangles, *viz.* That propoſed, and that aſſumed, being alike are proportional by the ſecond of the 6th. of *Euclid*. That is, (obſerving what ſide is given in the propoſed Triangle.) As any one ſide in the aſſumed Triangle, is to his correſponding ſide in the propoſed Triangle: So is any other ſide in the aſſumed Triangle, to his correſponding ſide in the propoſed Triangle. Hereby (comparing like ſides) every ſide in the propoſed Triangle may be exactly found, and this method fails not to give the true ſolution to three or four places, which I preſume is exact enough for the Marriners uſe; for which it is chiefly intended; but if the Operations be continued to two or three places of Decimals, the ſolution ſhall be exact enough for any uſe. And whereas in moſt Treatiſes of Trigonometry, the Doctrine of Right angled plain Triangles is uſually divided into ſeven Caſes; yet if they be diſtinguiſhed only by their data's they admit but of four, as follows, The firſt and ſecond whereof in which the angles are given, are only ſolvable by the foregoing Rules.

Note, That when the angles are given in degrees and minutes, thoſe minutes muſt be reduced into Decimal parts; ſo having premiſed thus much we ſhall proceed to;

The Firſt Caſe.

THE angles and one of the Legs being given, To find the Hypothenuſe and the other Leg.

Example.

In the Right angled plain Triangle ABC.

There is given, Fig. 1.

	d. m.	d.	
BAC	33—42 or	33,7	} AC and BC
AB	222		} required.

In

In order to the Solution hereof.

The first Operation is to find the Hypothenuse A c, and the greater Leg A b, in the assumed Triangle A b c, the angles thereof being the same as in the proposed Triangle ABC, *viz.* b A c 33,7, A c b 56,3, and the lesser Leg b c 1,0 *&c.*

b A c

33,7) 172,0 (5,1
350
13

5,1
5,1
51
255
squ. of the quotient 26,01
3 abated Remain. 23,01 (4,8
16
88) 701
704

Quotient is 5,1
doubled is 10,2
Root substr. 4,8
Rem. divided by 3) 5,4
Hypoth. A c is 1,8
Hypoth. doubl. is 3,6
Greater Leg A b is 1,5

The Second Operation, To find the Hypothenuse A C, and the lesser Leg B C which is required in the proposed Triangle A B C.

For the Hypothenuse A C say,

As A b : A B : : A c : A C.
1,5 : 222 : : 1,8 : 266,4 required.

222
1,8
1776
222
1,5) 399,6 (266,4
99
96
60

 For

For the Leg B C say,

As A b	:	A B	::	b c	:	B C
1,5	:	222	::	1,0	:	148 required.

1,0

1,5) 222,0 (148
72
120

ans-{ A C is 266,4 } which was
wer { BC is 148 } required.

Note, That if you require more exactness, the work must be continued to more places of Decimals; as in the Examples following.

The first Case and second Example.

In the Triangle A B C — Fig. 2.
There is given

	d.	m.		d.	
BAC	36	—48	or	36,8	AC and BC required.
A B	90				

The first Operation, To find the Hypothenuse A c and the greater Leg A b in the assumed Triangle A b c, the angles being the same with the proposed Triangle A B C, and the lesser Leg b c 1,00, *&c.*

b A c
d.
36,8)172,0 (4,67
2480
2720
144

4,67
4,67
3269
2802
1868
squ. of the Quot. 21,8089

3 abated Remain. 18,8089 (4,34
16
83) 280
249
864) 3189
3456

Quotient is 4,67
doubled is 9,34
Root substra. 4,34
Rem. divid. by 3) 5,00
Hypoth. A c is 1,67
Hypoth. doubl. is 3,34
Greater Leg A b 1,33

The

The Second Operation, To find the Hypothenuse A C, and the lesser Leg B C, which is required in the proposed Triangle A B C.

For the Hypothenuse A C say,

As Ab : AB : : Ac : AC
1,33 : 90 : : 1,67 : 113 required

90

1,33) 150,30 (113
133
170
1

For the Leg B C required say,

As Ab : AB : bc : BC
1,33 : 90 : 1,00 : 67,6 required.

1,00

1,33) 90,00 (67,59

1020

790

answer { A C is 113 / B C 67,6 } which was required.

The first Case Third Example.

In the Triangle A B C — Fig. 3.
There is given

	d.	m.	d.	
B A C	59	36	or 59,6	A C and B C required.
B C 212				

The first Operation, To find the Hypothenuse A c, and the greater Leg b c in the assumed Triangle A b c the angles being the same as in the Triangle A B C proposed, and the lesser Leg A b 1,00, &c.

Acbd.

30,4) 172,0 (5,658
5,658
2000
45264
1760 28290
88948
240 28290

squa.of the quot. 32,012964

3 substr. Rem, 29,012964 (5,386
25
103) 401
309
1068) 9229
8544
10766) 68564
64596
3968

Quotient is 5,658
doubled is 11,316
Root Substr. 5,386
Rem.divid.by 3) 5,930
Hypot. A c is 1,977
Hypot. doubl. is 3,954
greater Leg. b c is 1,704

The Second Operation, To find the Hypothenuse A C and the lesser Leg A B in the Triangle A B C proposed;

For the Hypothenuse A C Say,

As bc : BC : : Ac : AC
1,704 : 212 : : 1,977 : 245,9 or 246 required.

1977
212
3954
1977
3954

1,704) 419,124 (245,9
7832
10164
16440
1104

For

For A B the required Leg say.

	bc	:	BC	:	Ab	:	AB	
As	1,704	:	212	:	1,00	:	124,4	requir.

```
          1,00
1,704) 212,00 (124,4
        4160
        7520
        7040
         224
```

Answer $\left\{\begin{array}{l}\text{A C is 245,9 or 246} \\ \quad\quad\text{And} \\ \text{A B 124,4}\end{array}\right\}$ which was requi.

The Fourth Example of the First Case.

IN the Triangle A B C. Fig. 4.
There is Given

d. m. d.
A C B 39 — 23 or 39,38
And
A B 92. } A C & B C required.

The first Operation, To find the Hypothenuse A c, and greater leg b c in the assumed Triangle A b c: having the same Angles with the proposed Triangle A B C, and the lesser A b 1,00, &c.

A c b d.

39,38 (172,00 — 14480 — 26660 — 3032

4,368 × 4,368 = 34944, 26208, 13104, 17472

squ. of the Quotient 19,079424

3 abated Remain. 16,079424 (4,0099 or 4,010 *fere*
16
8009) 079424
72081
80189) 734300
721701
12599

Quotient is 4,368
doubl. is 8,736
Root substr. 4010
Rem.div.by 3) 4,726
Hypoth. Ac is 1,575
doubl. is 3,150
greater Leg b c is 1,218

The second Operation, To find the Hypothenuse A C, and the greater Leg B C in the proposed Triangle A B C;

For A C the Hypothenuse say.

As A b : A B : : A c : A C
1,00 : 92 : : 1,575 : 144,9 required.

1,575
92
3150
14175
144,9100

For

For B C the required Leg ſay.

As	Ab	:	AB	: :	bc	:	BC
	1,00	:	92	: :	1,218	:	112,1 *fere.*

$$
\begin{array}{r}
1{,}218 \\
92 \\
\hline
2436 \\
10962 \\
\hline
112{,}0156
\end{array}
$$

Anſwer { A C 144,9 And B C 112,1 *fere* } which was required.

Note, That for variety ſake, as alſo for better illuſtration of the aforementioned Rules, I have added more Examples in this Caſe, ſo ſhall give fewer in the following Caſe.

The Second Caſe.

THE Angles and the Hypothenuſe being given, To find the Legs;

The firſt Example.

In the Triangle A B C — Fig. 5.

There is given

AC 182

d. m. d.

BAC 41—42 or 41,7 } A B and B C required.

The firſt Operation, To find the Hypothenuſe Ac, and the greater leg Ab in the aſſumed Triangle A b c wherein the Angles are the ſame as is propoſed, and the leſſer Leg b c 1,00, &c.

b A c d. The Quotient is 4,125

41,7) 172,0 (4,125 doubled is 8,250
4,125 Root substr. 3,744
520
20625 Rem. divided by 3) 4,506
1030
8250
4125 Hypoth. Ac is 1,502
1960 16500
Hypoth. doubled is 3,004

Squ. of the Quot. 17,015625

greater Leg A b is 1,121

3 abated Remaind. 14,015625 (3,744
9
67) 501
469
744) 3256
2976
7484) 28025
28336

The Second Operation. To find the two Legs AB and B C in the Triangle A B C proposed.

For the greater Leg A B say,

As	Ac	:	AC	: :	Ab	:	AB
	1,502	:	182	: :	1,121	:	135,8 required.

1,121
182
364
182
182
1,502) 204,022 (135,8
5382
8762
12520
504

For

For the lesser Leg B C say,

	Ac	: AC	: :	bc	:	B C requir.
As	1502	: 182	: :	1,00	:	121,17 or (121,2

```
            1,00
1,502) 182,00 (121,17
        3180
        1760
        12580
        10780
```

Answer { A B is 135,8 And B C is 121,2 } which was required.

The Second Case, and Second Example.

In the Triangle A B C Fig. 6.
There is given

d. m. d.
B A C 54-18 or 54,3 And AC 193 } A B and B C required.

The First Operation, To find the Hypothenuse Ac, and the greater leg bc in the assumed Triangle Ac b, in which the Angles are the same as in the proposed Triangle A B C and the lesser leg A b 1,00 &c.

Acb

A c b
d.
35,7) 172,0 (4,818
2920
640
283

4,818
4,818
38544
4818
38544
19272

squ. of the quot. 23,213124

3 abated Remain. 20,213124 (4,496
16
84) 421
336
889) 8531
8001
8986) 53024
53916

Quotient is 4,818
doubled is 9,636
Root substr. 4,496
one third Rem. 5,140
Is the Hypoth. A c 1,713
Hypoth. doub. is 3,426
greater Leg b c is 1,392

The Second Operation, To find the Legs A B and B C in the Triangle A B C proposed;

For A B the lesser Leg say:

As A c : A C : A b : A B
1,713 : 193 : 1,00 : 112,67 required.
1,00

1,713) 193,00 (112,67
2170
4570
11440

For

For B C the greater Leg say,

	Ae		AC		bc		BC
As	1,713	:	193	: :	1,392	:	156,8 required.

1392
193
————
4176
12528
1392
————
1,713) 268,656 (156,83
————
9735
————
11706
————
14280

answer
AB is 112,67 and
BC 156,8
which was required.

The Third Case.

THE Legs being given, To find the Hypothenuse and the angles.

First Example.

In the Triangle A B C. Fig. 7.
There is Given

A B 81 } A C and B A C
B C 54 } are required.

For the Solution of this Case.
First find the Hypothenuse A C by Extraction of the square-root, to perform which,

The Rule is.

Square the Legs severally, and add their Squares together, then Extract the Square Root of that Sum, which Root is the Hypothenuse required.

The

The Operation.

Leg A B is	81	Leg B C is	54
Squared	81	Multipl. in it self	54
	81		216
	648		270
The square of A B is	6561	The Squ. of B C is	2916
The Squ. of B C add	2916		
The square of A C is	9477 (97,3 A C required.		
	81		
	187) 1377		
	1309		
	1943) 6800		

Secondly, Having found the three Sides in any Right angled plain Triangle, the angles may be thus found, *viz.* get the Sum of the Hypothenuse, and half the greater Leg; Then the proportion is

As the Sum of the Hypothenuse and half the greater Leg.
Is to the lesser Leg. So is 86
To the angle opposite to the lesser Leg.

The Operation.

The Hypothenuse A C is	97,3
Half of A B the greater Leg is	40,5
The Sum is	137,8

The

The Proportion to find B A C the lesser angle opposite to the less Leg. BC

As 137,8 : 54 : : 86 : 33,7 : BAC required.

```
                 d.
As 137,8 : 54 : : 86 : 33,7 : BAC required.
                   54
                  ----
                  344
                 430
                 ----    d.      d.  m.
       137,8) 4644 (33,7 or 33--42 is the ang.BAC
              ------
               5100
               ----
               9660
               ----
                 14
```

Answer { AC is 97,3 d. m. } which was required.
Answer { BAC 33,7 or 33--42 } which was required.

The Second Example of the Third Case.

IN the Triangle A B C Fig. 8.
There is given
A B 61 } A C and A C B
B C 87 } required.

The First Operation, To find A C by the square Root.

```
The Leg A B is          61          The Leg B C 87
 Which squared          61      multipl. in it self 87
                       ----                      -----
                        61                        609
                       366                        696
                       ----                      -----
The squ. of A B is     3721      The squ. of B C is 7569
The squ. of B C add    7569                      -----
                       -----
squ. Hypoth. A C  11290 (106,2 is AC required.
                   1
                  ----
            206)  1290
                  1236
                  ----
           2122)  5400
```

The

The Second Operation, To find A C B.

The Hypothenuse A C is	106,2
The half of the greater Leg BC	43,5
The sum is	149,7

The Proportion is,

As 149,7 : 61 : : 86 : 35,04 d. or 35–02 d. m. the angle ACB required.

```
        86
        61
      ----
        86
       516
      ----        d.
149,7) 5246 (35,04
      ----
       7550
      ----
       6500
      ----
        512
```

Answer { A C is 106,2 and / A C B 35,04 d. or 35,02 d. m. } which was required.

The Fourth Case.

THE Hypothenuse and one of the Legs being given. To find the other Leg and the angles.

The First Example.

In the Triangle A B C.
There is given. Fig. 9.

A C 112 } A B and A C B
A C 57 } required.

For

For the Solution hereof: First find A B the Leg required by the square Root. To effect which,

The Rule is,

Square the Hypothenuse and the given Leg severally, and from the square of the Hypothenuse, substract the square of the Leg. Then Extract the square Root of the Remainder which Root is the Leg required.

The Operation.

The Hypothenuse A C is	112	The given Leg B C	57
Which mult. in it self	112	Which square	57
	224		399
	112		285
	112		
The squ. of A C is	12544	The squ.of B C	3249
The squ. of B C substr.	3249		

The squ.of the Leg A B is 9295 (96,4 is A B required.
81

186) 1195
1116

1924) 7900
7696

204

Secondly, Having the Hypothenuse and the two Legs, find the angle opposite to the lesser Leg as before in the 3d. Case.

The Operation to find B A C.

The Hypothenuse A C is	112
Half of the greater Leg A B is	48
The summ is	160

d.
As 160 : 57 : : 86 : 30,6

```
      86
      57
     ----
     602
    430
     ----             d.
160) 4902,0 (30,6
     ----
     1020        answer
     ----   { AB is 60.4 and      d. m. }
     60     { ACB [illegible] or 59 24  }
                 which was required.
```

The Second Example of the Fourth Case.

IN the Triangle A B C Fig. 10.
There is given
A C 119 } B C and A C B re-
A B 87 } quired.

The first Operation, To find the Leg B C, by the square Root, as in the foregoing Example.

```
A C the Hypothenuse is 119      The Leg AB is 87
         which square  119         which square 87
                      -----                    -----
                      1071                      609
                      119                      696
                     119                       -----
                     -----       squ. of A B   7569
The square of A C is  14161                    -----
The square of A B substr. 7569
                     -----
Squr. of the Leg B C is 6592 (81,2 is BC required.
                        64
                       -----
                  161)  192
                        161
                       -----
                 1622) 3100
```

The

The Second Operation to find BAC.
The Hypothenuse A C is 119
Half of A B the greater Leg is 43,5

The Sum is 162,5 B C is 81,2

As 162,5 : 81,2 : : 86 : 43 d.

	81,2
	86
	4872
	6496
162,5)	6983,2 (43 d. fere, is B A C required.
	4832

Answer { B C is 81,2 and
A C B 47 d. fere } which was required.

Thus much for our Essay and method offered for the performance of the Doctrine of Right angled plain Triangles, by Natural Arithmetick (without the Tables of Natural Sines and Tangents) wherein all the Examples here added, where the Operations are continued to two or three places of Decimals. The Solution thereby seldome differs above three or four Centisms, and sometimes less, from their Logarithmical Solutions, which evidently manifests the certainty of this method before any hitherto made publick. Now follows the Doctrine of Oblique angled plain Triangles.

Arithmetical Trigonometry,

OR,

The Doctrine of Oblique Angled Plain Triangles, performed by Natural Arithmetick.

IN which it is observable, That the Doctrine of Oblique angled plain Triangles is usually divided into five Cases, but being distinguished only according to their Data's, admit of no more then these four, which hereafter follow; of which the three first wherein the angles are given may be resolved by those Rules already delivered for Right angled plain Triangles. In order whereunto the Oblique angled plain Triangle proposed, must first be reduced into two Right angled plain Triangles, by letting fall a perpendicular, which must always fall from the end of a given side, and opposite to a given angle, the angle joyning to the given side, so will the given side become the Hypothenuse and the given angle the angle at the Base. That wherein is given the side and angle may be called the first Triangle, the other the second, this being premised we shall proceed to the first Case.

The First Case.

TWo sides and an angle opposite to one of them being given, To find the third side and either of the other angles.

First Example.

In the Oblique angled plain Triangle A D E.

There is given.

A D 68 *d.*
D A E 41,2 } A E and A E D required. Fig. 11.
D E 79.

Note, That in all questions pertaining to this Case, the perpendicular being let fall according to the former Rules, always falls upon the required side; therefore if the greatest side be required, it falls within, as in this first Example. If either of the other two sides without the Triangle, as in the second Example, & then the side upon which it falls must be continued, and for the Solution thereof there is always the Hypothenuse and the angles known, in the first Triangle to find the Legs, as in the second Case of Right angled plain Triangles, which done, there will be the Hypothenuse and one of the Legs given in the second Triangle, to find the other Leg and the angles, by the fourth case of Right angled plain Triangles.

In this Example aforesaid; ADB is the first Triangle, wherein the Hypothenuse AD is 68, & the angle BAD 41,2 *d.* To find the Legs AB and BD.

Therefore the first Operation is for the assumed Triangle Adb, corresponding the first Triangle ADB, as having the same angles therewith, and the lesser Leg b d 1,00 &c. To find the Hypothenuse A d and the greater Leg A b.

Quotient is 4,175.

d.
41,2) 172,0 (4,175
4,175
720
20875
3080
29225
4175
196
16700

squ. of the Quotient 17,430625

3 abated Remain. 14,430625 (3,799
9
67) 543
469
749) 7406
6741
7589) 66525

doubl. is 8,350
Root substr. 3,799
$\frac{1}{3}$ of the Rem. 4,551
Hypoth. Ad is 1,517
doubl. is 3,034
greater Leg Ab is 1,141

The second Operation, To find the Legs A B and B D, in The first Triangle A B D

For A B the greater Leg say.

As Ad : AD :: Ab : AB
1,517 : 68 :: 1,141 : 51,14 required.
68
9128
6846
1,517) 77588 (51,14 AB
1738
221

For

For B D the lesser Leg say,

	Ad		AD		bd		BD
As	1,517	:	68	: :	1,00	:	44,8 required.
			1,00				

1,517) 68,00 (44,8

7320

1252

Then in the second Triangle E D B.
There is given,
D E the Hypothenuse 79 } B E and B E D
and D B the perpendicular 44,8 } required.

First, Find BE by extraction of the Square Root, as in the fourth Case of Right angled plain Triangles.

The Operation.

The Hypothenuse D E is 79
Which square 79

711
553

The Leg BD is 44,8
which square 44,8

3584
1792
1792

Squ. of the Hypothenuse 6241
Squ. of the Leg, B D 2007,04 Squ. of BD 2007,04

The squ. of the Leg B E 4233,96 (65,67 is B E requir.
36

125) 633
625

13007) 0896

 Secondly,

Secondly, Having found the three Sides, find the angle B E D opposite the lesser Side B D, as in the third and fourth Cases of Right angled plain Triangles.

The Operation.

The Hypothenuse D E is 79
Half of the greater Leg B E is 32,5

The sum is 111,5   B D is 44,8

As 111,5 : : 44,8 : 86 : 34,55. (d.)

```
        44,8
          86
       -----
        2688
       3584
       ----------         d.      d.  m.
111,5) 3852,8(34,55 or 34--33 BED.
       ------
        5078
       ------
         6180
       ------
          695
```

Unto A B the base in the first Triangle 51,14
Add BE the base in the second Triangle 65,07

The sum is A E which was required 116,21

Answer A E is 116,2 and
A E D 34,55 (d.) or 34—33 (d. m.) } which was required.

The

The First Case and Second Example.

IN the Triangle A D E Fig. 12.
There is given.

D E 82 *d.*
A D E 106,3 } AD and DAE required.
A E 120

In this Example the perpendicular DE (according to the foregoing Rules) falls without the Triangle, and the required Side A D upon which it falls is continued, then is D E B the first Triangle; wherein is known the Hypothenuse D E 82, and the angles, *viz.* B D E, 73,7*d.* (the Complement of ADE to 180 d.) and D EB 16,3, d. To find the Legs E B and D B: So that preparatory to Solution,

The first Operation is to find D e the Hypothenuse, and b e the greater Leg in the assumed Triangle, Dbe, which is equiangular to the first Triangle D E B, and the less Leg thereof D b 1,00 &c.

Quotient is 10,552
d.
16,3)172 (10,552 doubled is 21,104
10,552 Root substr. 10,409
920
21104 one third Rem. 10,695
850 52760
52760 Is the Hypoth. D e 3,565
35 105520
Hypoth. doub. is 7,130
squ. of the quot. 111,344704
greater Leg b e is 3,422
. . . . .
3 abated Remain. 108,344704 (10,409
1
204) 0834
816
20809) 184704

The

The Second Operation, To find the Legs B E and B D in the first Triangle D E B.

For B E the greater Leg it holds.

```
As De    : DE : be    : B E
   3,565 : 82 : 3,422 : 78,7 required.
                82
             ------
              6844
             27376
             ------
   3,565) 280,604 (78,71
           ------
            31054
           ------
            25340
           ------
              385
```

For BD the lesser Leg, say.

```
As De      DE      Db       DB
   3,565 : 82    : 1,00   : 23 required.
           1,00
         -------
   3,565) 82 00 (23
          10709
         -------
              5
```

Then in the second Triangle AEB. There is given,

AE the Hypothenuse 120 } To find A B and
and B E, the perpendicular 78,7 } B A E.

The first Operation is to find A B by the Extraction of the square Root.

The

The Hypothenuse AE	120	The Leg B E	78,7
Which square	120	Which square	78,7
	2400		5509
	120		6296
			5509
The Square of A E	14400		
The Square of B E substr.	6193,69		6193,69

Remains the Squ. of A B 8206,31 (90,59 or 90,6 *fere* is
81 (A B required.
1805) 10631
9025
1810) 160600

The second Operation (having the three Sides) to find the angle B A E opposite to the less Leg B E.

The Hypothenuse A E is 120
Half of A B the greater Leg is 45,3

The Sum is 165,3 B E is 78,71

As 165,3 : 78,71 : : 86 : 41 *d. fere* is BAE required.
86

47226
62968

165,3) 6769,06 (41 *fere*

15,706

From AB the base in the second Triangle 90,6
Substract DB the base in the first Triangle 23,

There Remains A D which was required 67,6

Answer AD is 67,6 and DAE 41 *d. fere* } which was required.

Note,

Note, That in all questions appertaining to this Case, *viz.* Wherein two sides and an angle opposite to one of them is given, the angle opposite to the other given side is sometimes dubious, and may be either acute or obtuse; that is, if the given angle be opposite to the greater of the given sides, the angle opposite to the other given side is acute, as in the two preeceding Examples.

But if the given angle be opposite to the lesser of the given sides, then the angle opposite to the other given side is doubtful, and may be either acute or obtuse;

And when the Case is doubtful, the third side is also various, according as the angle opposite to the other given side is required, whether acute or obtuse, as in the two Examples following.

The Third Example of the First Case.

IN the Triangle A D E — Fig. 13.
There is given

	d.	
D A E	35,4	AD and ADE required obtuse.
A E	108	
D E	70	

In this Example, the angle ADE being required obtuse, the Solution falls in the Triangle A′ D E.

And then (according to the foregoing Rules) the perpendicular EB falls without, upon the required side A D continued, so that AEB is the first Triangle, wherein the Hypothenuse A E and the angles are known, to find the Legs AB and B E; Therefore

The first Operation is for the assumed Triangle A e b equiangular to the first Triangle ABE, the lesser Leg b e being 1,00 &c. To find A E the Hypothenuse, and A b the greater Leg.

35,4)

The Quotient is 4,859

d.

35,4) 172,0 (4,859

3040

2080

310

4,859	doubled is 9,718
4,859	Root subſtr. 4,540
43731	Rem. divided by 3) 5,178
24295	
38872	Hypoth. Ae is 1,726
19436	
	Hypoth. doubled is 3,452

Squ. of the Quot. 23,609881

greater Leg A b is 1,407

3 abated Remaind. 20,609881 (4,54 fere.

16

85) 460

425

904) 35988I

The Second Operation. To find the Legs AB and B E d in the Triangle A B E.

For A B the greater Leg, the proportion is,

As Ae : AE : : Ab : AB.
1,726 : 108 : : 1,407 : 88 required.

108

11256

14070

1,726) 151956 (88,04

13876

6800

For

For B E the leſſer Leg it is,

As Ac : AE : : be : BE
1,726 : 108 : : 1,00 : 62,57 required,

```
         108
        1,00
       ------
1,726) 108,00 (62,57
       ------
        4440
        ----
         9880
         ----
         1250
```

Then in the ſecond Triangle DEB. There is given

DE the Hypothenuſe 70 } To find B D and
and BE the perpendicular 62,57 } B D E

The firſt Operation, To find BD by the Square-Root.

```
The Hypothenuſe D E  70          The Leg B E is 62,57
which Square         70             which ſquare 62,57
                    ----                         -----
Squ. of the Hypoth. 4900                         43799
Squ. of the Leg B E 3915                        31285
                    ----                        12514
Squ.of the Leg BD is 985, (31,4=B D            37542
                     9                     -----------
                    ----                     3915,0049
                 61) 85
                     61
                    ----
                624) 2400
```

The

The second Operation to find the angle BED opposit the less Leg B D.

The Hypotheause D E is 70
Half of B E the greater Leg is 31,3

The Sum is 101,3 B D is 3

				d.
As 101,3 :	31,4 :	:	86 :	26,66
	86			
	1884			
	2512			

d. d. m.
101,3) 2700,4 (26,66 or 26—39 is B E D requir
6744 — the Complement wherec
6660 — 90 d. is 63 d. 21 m. BDE
528 — the Complement of BE 180 d. is 116 d. 39 m obtuse angle A D E requ

From A B the base in the first Triangle 88,04
Substr. DB the base in the second Triangle 31,40

There Remains AD which was required 56,64

Answ. A D is 56,6 and d. m. } which was required.
ADE 116,65 d. or 116,39 }

If the angle opposite to the given side A E, had be quired acute, the Solution would have fallen in the gle AdE, wherein AdE would have been the angle a the third side required, thus found, viz. the perpen E B (agreeable to the foregoing Rules) would hav within. And A B E would have been the first Tria

find AB & BE as before as alſo BdE (equal BDE is the ſecond Triangle, therein to find the angle BDE (equal BDE 63 *d.* 21 *m.*) and the ſide Bd (equal BD 31,4) which done. Then,

Unto AB the baſe in the firſt Triangle	88,04
Add Bd the baſe in the ſecond Triangle	31,4
The ſum is Ad which was required	119,44

The Fourth Example of the Firſt Caſe.

IN the Triangle ADE. Fig. 14.
There is Given

	d.	
AED	37,6	AE and DAE required acute.
AD	63	
DE	91	

In this Example, becauſe the angle DAE is required acute. The Solution falls in the Triangle ADE, and (according to the foregoing Rules) the perpendicular DB falls within the Triangle, and then EDB is the firſt Triangle, in which the Hypothenuſe DE, and the angles are known to find the Legs EB and DB, but

The Firſt Operation, is for the aſſumed Triangle Edb equal angled to the firſt Triangle EDB, and the leſs Leg db, 1,00 To find the Hypothenuſe Ed and the greater Leg Eb.

37,6)

Quotient is 4, 574

d.

37,6) 172,0 (4,574 doubled is 9, 148

4,574 Root Substr. 4, 233

2160

18296 Rem.divid.by 3)4,915

2800 32018

22870 Hypot. E d is 1, 638

1680 18296

Hypot. doubl. is 3, 276

squa.of the quot. 20,921476

greater Leg.E b is 1, 298

3 abated Rem. 17,921476 (4,233

16

82) 192

164

843) 2814

1529

8463) 28576

The second Operation, To find the Legs E B and D B in the first Triangle E D B.

First, For E B the greater Leg it is,

Ed		ED		Eb	EB
As 1,638	:	91	: :	1,298	72,1 required.

91

1298

11682

1,638) 118,118 (72,11

3458

1820

182

Secondly, For the lesser Leg D B say,

E d E D d b DB
As 1,638 : 91 :: 1,00 : 55,56 required.

1,00

1,638) 91,00 (55,56

9100

9100

9100

Then in the second Triangle ADB, There is given,

The Hypothenuse A D 63 } To find A B and
& BD the perpendicular 55,56 } B A D.

The first Operation, To find A B by the square-Root.

The Hypothenuse AD is 63. The Leg BD is 55,56
Which multipl. in it self 63 Which square 55,56

189 33336
378 27780
  27780
  27780

The squ. of the Hypoth. 3969,00
The squ. of the Leg BD is 3086,91 3086,9136

The squ. of the Leg AB is 882,09 (29,7 is A B required.

4

49) 482

441

587) 4109

4109

0

The

The second Operation is (having the three sides) to find the angle A D B opposite the less Leg A B.

The Hypothenuse AD is 63
Half of DB the greater Leg is 27,78

The Sum is 90,78 AB is 29,7

As 90,78 : 29,7 : : 86 : 28,14 deg. ADB

```
          29,7
          86
         ------
         1782
        2376
        ------        deg.    d. m.
90,78)  2554,2 (28,14 or 28—08 ADB required the
        ------
         73860
        ------
         12360
        ------
          9280
```

Complement whereof to 90 *d.* is 61 *d.* 52 *m.* the angle D A E required Acute.

Unto EB the base in the first Triangle	72,1
Add AB the base in the second Triangle	29,7
The sum is A E the side required	101,8

In this Example, if the angle opposite to the given side D E, had been required obtuse; the Solution would have been in the Triangle a D E, in which D a E would have been the angle, and a E the third side required. And then (according to the former Rules) the perpendicular D B falls without the Triangle, and EDB would have been the first Triangle, To find E B and DB as before, and aDB (equal ADB) the second Triangle, therein to find the angle BaD (equal BAD) 61 *d.* 52 *m.* and the side a B (equal AB) 29,7. which done the obtuse angle D a E and the side a E is thus had, *viz.*

From EB the baſe in the firſt Triangle 72,1
Subſtract a B the baſe in the ſecond 29,7

Remains the ſide aE required 42,4

And the Complement of B a D 61 d.—52 m. to 180 d. is 118 d.—08 m. the angle D a E required obtuſe.

Therefore if the angle at A be required acute.

The anſwer is DAE 61 d. 52 m. and
AE is the third ſide 101,8

But if the angle at A be required obtuſe.

The anſwer is D a E 118 d.-08 and
a E the third ſide is 42,4

The Second Caſe.

THE angles and one ſide being given to find the other two ſides.

The Firſt Example.

In the Oblique plain Triangle A D E. There is given

Fig. 15

	deg.		d. m.	
ADE	118,8	or	118—48	A E and D E required.
DAE	26,7	or	26—42	
And AD	85,5			

Note, That in all queſtions pertaining to this Caſe the the perpendicular (agreeable to the former Rules) will always fall upon one of the required ſides, if either of the leſ-

ſer

ſer ſides be given it falls within, as in this firſt Example ; but if the greateſt ſide be given it falls without the Triangle, upon either of the required ſides being continued, as in the ſecond example : and towards Reſolution thereof, there is always the angles and the Hypothenuſe given in the Firſt Triangle,to find the Legs as in the ſecond Caſe of Right angled plain Triangles : and in the ſecond Triangle, you always have the angles and one of the Legs known, to find the the Hypothenuſe and other Leg,as in the firſt Caſe of Right angled plain Triangles.

In this Example ADB is the firſt Triangle wherein the Hypothenuſe AD is 85,5 and the angle BAD 26,7 *deg.* To find the Legs AB and DB, and in order to the Solution thereof.

The firſt Operation, is for the aſſumed Triangle Adb equiangular to the firſt Triangle ADB,and the leſſer Leg db 1,00 &c. to find the Hypothenuſe A d, and the great Leg A b ;

		Quotient is 6,442
d.		
26,7) 172,0	(6,442	doubl. is 12,884
	6,442	Root ſubſtr. 6,205
1180		
	12884	$\frac{1}{3}$ of the Rem. 6,679
1120	25768	
	25768	Hypoth. Ad is 2,226
52	38652	
		doubl. is 4,452
ſqu. of the Quotient	41,499364	greater Leg Ab is 1,990

3 abated Remain. 38,499364 (6,205

36

122) 249

244

12405) 59364

The

The second Operation, To find the Legs A B and D B, in The first Triangle A D B

First For A B the greater Leg it holds.

As Ad	AD	Ab	AB
2,226 :	85,5 :	: 1,99 :	76,43 required.

85,5
1,99
———
7695
7695
855
———
2,226) 170,145 (76,43
———
14325
———
9690
———
786

Secondly, For BD the lesser Leg the proportion is,

As Ad :	AD :	: bd :	BD
2,226 :	85,5 :	: 1,00 :	38,41 required.

85,5
1,00
———
2,226) 85,500 (38,41
———
18720
———
9120
———
216

Then in the second Triangle EDB. There is given

BED 34,5d. } DE the Hypothenuse, and B E the Base
DB 38,4 } required.

The first Operation is, for the assumed Triangle Egb equiangular to the second Triangle EDB, and the lesser Leg thereof gb 1,00 and to find the Hypothenuse Ec and the greater Leg Eb.

34,5)

The Quotient is 4,986

d.

34,5) 172,0 (4,986 doubled is 9,972

3400 4,986 Root substr. 4,675

2950 29915 Rem. divided by 3) 5,297

190 39888

44874 Hypoth. Ec is 1,766

19944

Hypoth. doubled is 3,532

Squ. of the Quot. 24,860196

greater Leg E b is 1,454

3 abated Remaind. 21,860196 (4,675

16

86) 586

516

927) 7001

6489

9345) 51296

46725

4571

The Second Operation. To find the Hypothenuse DE and B E the base in the second Triangle E,D,B.

First; For D E the Hypothenuse say,

As bg : BD : : Ec : DE

1,00 : 34,8 : : 1,766 : 67,81 required.

38,4

7064

14128

5298

1,00) 67,8144 (67,81

Secondly, For B E the base it holds,

	bg		BD		Eb		EB
As	1,00	:	38,4	:	1,454	:	55,84 *fere.*

38,4

5816
11632
4362

1,00) 55,8336 (55,84

Unto A B the base in the first Triangle 76,43
Add EB the base in the second Triangle 55,84

The sum is A E required 132,27

Answer { A E is 132,3 *fere* and / D E is 67,8 } which was required.

The Second Example of the Second Case.

IN the Triangle A D E — Fig. 15.
There is given

A E 124
A E D 39,35 *d.* } AD and DE required.
ADE 112,5 *d.*

In this Example the perpendicular (agreeable to the foregoing Rules) falls without the Triangle, and may either fall from A upon D E continued, or from E upon A D continued, as it is here done from E, and AEB is the first Triangle, wherein

wherein A E the Hypothenuse is 124, and the angle BAE 28,15 *deg*. To find the Legs A B and B E, Therefore

The first Operation is for the assumed Triangle Aeb equiangular to the first Triangle AEB and be the lesser Leg 1,00, &c. To find the Hypothenuse A e and A b the greater Leg.

```
d.
28,15)172,00 (6,11        6,11
       3100               6,11
       2850               611
        35                611
                          3666
sq. of the Quot.       37,3321
3 abated Remain.       34,3321 (5,86
                       25
                 108)  933
                       864
                 1166) 6921
```

```
The Quotient is       6,11
doubled is           12,22
Root substra.         5,85
Rem. divid. by 3)     6,36
Hypoth. A e is        2,12
Hypoth. doubl. is     4,24
Greater Leg A b       1,87
```

The Second Operation, To find the Legs AB and B E the first Triangle A E B.

First for A B the greater Leg say.

```
    Ae        AE       Ab        A B
As 2,12  :   124   :  1,87   :  109,4 required.
             1,87
             868
            992
           124
   2,12) 231,88 (109,38
         1988
          800
          164
```

Secondly, For B E the lesser Leg it holds.

As Ae 2,12 : AE 124 : : be 1,00 : BE 585, required.

```
        1,00
  2,12) 124,00 (58,5
        1800
        104
```

Then in the second Triangle D E B.

There is given,

B D E 67,5 d. the Complement of ADE 112,5 d. to 180 d.) and B E the perpendicular 58,5, To find D E the Hypothenuse and DB the base; So that,

The first Operation is for the assumed Triangle Dca equal-angled to the second Triangle D E B, and D a the lesser Leg 1,00 To find ac the greater Leg, and Dc the Hypothenuse, Dca the angle opposite to Da the lesser Leg is 22,5 *deg.*

```
 d.                                      Quotient is 7,644
22,5)172,0         (7,644                doubled is 15,288
                    7,644                Root substr. 7,445
     1450
                    30576           Rem.divid.by 3) 7,843
     1000           30576
                   45864             the Hypoth.D c is 2,614
     1000          53508
                                       Hypoth.doub. is 5,228
Squ. of the quot.  58,430736
                                    greater Leg a c is 2,416

Abated Remains 55,430736 (7,445
               49
          144) 643
               576
         1484) 6707
               5936
        14885) 77136
```

The

The Second Operation to find the Hypothenuse D E and the lesser Leg D B in the second Triangle DEB.

First, for the Hypothenuse D E it is.

	ac		BE			Dc		DE required,
As	2,416	:	58,5	:	:	2,614	:	63,3

```
                      2,614
                       58,5
                      -----
                      13070
                     20912
                    13070
                   --------
        2,416) 152,9190 (63,3 fere
                   7959
                   ----
                   7110
```

Secondly, For the Leg D B the proportion is,

	ac		BE			Da		DB
As	2,416	:	58,5	:	:	1,00	:	24,2 required,

```
              58,5
              1,00
             ------
   2,416) 58,500 (24,21
             ------
             10180
             ------
              5160
             ------
               328
```

From AB the base in the first Triangle 109,4
Substract DB the base in the 2d. Triangle 24,2

There Remains A D which was required 85,2

Answer DE is 63,3 and } which was re-
AD 85,2 } quired.

The Third Case.

TWO Sides and the angle between them, being Given, To find the third side, and the other angles.

First Example.

In the Triangle A D E. There is given.

A E 125
A D 78 } D E, A D E and AED required. Fig.17
D A E 31.4 d.

In all Questions that falls under the consideration of this Case, the perpendicular (by the former Rules) alwayes falls upon one of the given sides: and if the given angle be acute it may fall either within or without the Triangle: but if the given angle be obtuse, it falls without upon either of the given sides being continued. Then in the first Triangle you have always the Hypothenuse and the angles given to find the Legs; and in the second Triangle there is the Legs known, to find the Hypothenuse and angles, as in the third Case of Right angled plain Triangles;

In this Example the given angle being acute the perpendicular falls either within or without the Triangle, here I have let it fall within. And ADB is the first Triangle in which the angle BAD is 31.4 *d.* and Ad the Hypothenuse 78. To find the legs AB and BD so that towards the Solution hereof,

The first Operation is for the assumed Triangle Adb equiangular to the first Triangle ADB, and the lesser Leg db 1,00 &c. To find the Hypothenuse A d and the greater leg A b.

31.4)

d.		The Quotient is 5,47
31,4) 1720	(5,478	
	5,478	being doub. is 10,95
1500		Root substr. 5,19
	43824	
2440	38346	Rem. divid.by 3) 5,75
	21912	
242	27390	Hypoth. A d is 1,92
squ. of the Quot.	30,008484	Hypoth.doub. is 3,84
		greater leg Ab is 1,63
3 abated Remains	27,008484 (5,197	

25

101) 2,00
101

1029) 9984
9261

10387) 72384

The second Operation to find the legs AB and BD in the first Triangle ABD.

First, for AB the greater leg say.

	Ad		AD			Ab		AB
As	1,92	:	78	:	:	1,638	:	66,5 required.

1,638
78

13104
11466

1,92) 127,764(66,54

1256

1044

840

Fi-

From AE the whole base in proposed Triangle 126,00
Substract A B the base in the first Triangle 66,54

There Remains EB the base in the second Triang. 59,46

Secondly, for BD the lesser Leg it holds.

	Ad		AD		bd	BD	
As	1,92	:	78	: :	1,60	40,6	required.

1,00
1,92) 78,00 (40,62
1200
480

Then in the second Triangle EBD. There is given

DB the perpendicular 40,62 and } To find DE the Hypothe-
EB the base 59,46 } nuse and the angles.

The first Operation to find DE by the square-Root.

The Leg E B is 59,46 The Leg DB 40,62
Which square 59,46 Which square 40,62

35676 8124
23784 24372
53514 162480
29730

1659,9844

The Squ. of E B is 3535,4916
The Squ. of DB added 1659,9844

The Squ. of DE is 5195,4760 (72,09 *fere* DE required.
49

142) 295
284

14409) 114760

The

The second Operation is (knowing the three sides.) To find the angle BED opposite the lesser leg BD.

The Hypothenuse DE is	72,1
Half of BE the greater Leg is	29,7
The sum is	101,8

The lesser Leg BD is 40,6

As 101,8 : 40,6 : : 86 : 34,3 *d.*

```
        40,6
          86
        ----
        2436
       3248
       -----                d.            d.  m.
101,8) 34916 (34,3 fere or 34—18 the angle
       -----         BED required the Comple-
        4376         ment whereof to 90 d. is
        ----         55 d. 42 m. the angle BDE.
        3040
```

	d. — *m.*
Then unto ADB in the first Triangle	58—36
Add EDB in the second Triangle	55—42
Sum is the angle ADE which was required	114—18

Answer { DE is 72,1 AED 34—18 *d. m.* or 34,3 *d.* and ADE 114—18 *d. m.* or 114,3 *d.* } which was required.

The

The Second Example of the Third Case.

IN the Triangle A D E. Fig. 18.
There is Given

AE 122 }
DE 75 } AD, ADE and DAE required
AED 33,4d. }

In this Example because the given angle is acute, the perpendicular (agreeing with the former Rules) may fall either within or without the Triangle as in the last Example, but for variety sake I have here let it fall without, so that EAB becomes the first Triangle, in which the angle AEB is 34,4 d. and the Hypothenuse EA 122. To find the legs EB and BA, therefore,

The first Operation is for the assumed Triangle E a d corresponding the first Triangle EAB, a d the lesser leg being 1,00 to find Ea the Hypothenuse and Ed the greater leg.

d.
34,4) 172,0 (5
00

5
The Squ. of the quot. 25
3 abated Remain 22 (4,69
16
86) 600
516
929) 8400
8361
39

The quotient is 5,00
being doub. is 10,00
The Root substr. 4,69
Rem. divid. by 3) 5,31
The Hypoth. Ea is 1,77
Hypoth. doub. is 3,54
The greater Leg Eb is 1,46

The

The second Operation to find the Legs E B and BA in the first Triangle E A B.

First; For E B the greater Leg it is;

	Ea		EA		Eb		EB
As	1,77	:	122	: :	1,46	:	100,6 required.
			1,46				

732
488
122

1,77) 178,12 (100,64 fere.

1120

58

Secondly, For A B the lesser Leg it holds.

	Ea		EA		ab		AB
As	1,77	:	122	: :	1,00	:	68,9 required.
			1,00				

1,77) 122,00 (68,93 fere.

1580

1640

47

From EB the base in the first Triangle	100,64
Substract the given side DE	75,00
The Remains D B the base in the 2d. Triangle	25,64

Then in the second Triangle DAB. There is given.

A B 68,93 } To find A D and
D B 25,64 } B A D.

The first Operation to find AD by the square Root.

The Leg A B is	68,93	The Leg D B is	25,94
Which mult. in it self	68,93	Which square	25,64
	20979		10256
	62037		15384
	55144		12820
	41358		5128
The Squ. of A B is	4751,3449		657,4096
The Squ. of DB is	657,4096		

The Squ. of A D is 5408,7545 (73,54 is A D required.
49

143) 508
429

1465) 7975
7325

14704) 65045
58816

6229

The second Operation is having the three sides to find the angles.

The Hypothenuse A D is	73,54
Half of the greater Leg A B is	34,46
The Sum is	108,00

The lesser Leg D B is 25,64

As

As 108 : 25,64 : : 86 : 20,4 deg. BAD.

```
        25,64
           86
        -----
        15384
        20512
      -------          d.       d.  m.
108) 2205,04 (20,4 or 20--24 is BAD the Com-
     -------          plement of which to 90 d. is
         450          69 d. 36 m. the angle ADB.
        ----
          18
```

The Complement of ADB to 180 *d.* is the angle ADE 110 *d.* 24 *m.* which was required.

Answer { AD is 73,5 ADE 110,4 *d.* or 110--24 *d. m.* and DAE 35,2 *d.* or 35--12 *d. m.* } which was required.

The Third Example of the Third Case.

In the Triangle A D E.
There is Given Fig. 19.

	d.	
ADE	109,3	AE, DAE and AED required.
AD	86	
DE	73	

In this Example (the given angle being obtuse) the perpendicular (agreeable to the former Rules) falls without the Triangle upon either of the given sides continued, here I have let it fall from A upon DE continued, hereby DAB be-

comes the first Triangle wherein is given A D the hypothe-86, and ADB 70,7 *d.* (the complement of A D E to 180 *d.*) To find the legs AB and DB, therefore,

The first Operation is for the assumed Triangle Dab, equal angled with the first Triangle DAB, and the lesser leg Db 1,00 and to find Da the hypothenuse, and a b the greater leg thereof.

```
                                              Quotient is    8, 912
 d.                                                         --------
19,3) 172,0          (8,912                    doubled is   17, 824
      -----           8,912                   Root Substr.   8, 742
       1760          ------                                 --------
      -----           17824                Rem.divid.by 3)9,082
        230           8912                                  --------
      -----           80208                  Hypot. a D is 3, 027
         37           71296                                 --------
      -----          ------                Hypot. doubl.is  6, 054
squa.of the quot.   79,423744                               --------
                    ---------            greater Leg. a b is 2, 858

3 abated Rem.   76,423744 (8,742
                64
               -----
            167)1242
                1169
               -----
           1744) 7337
                 6976
                -----
          17482) 36144
                 -----
```

The

The second Operation, To find the Legs A B and D B in the first Triangle D A B.

First, For A B the greater Leg the proportion is,

	aD		AD		ab	AB
As	3,027	:	86	: :	2,858	81,2 required.

86

17148
22864

3,027) 245,788 (81,2 fere

3628

6010

Secondly, For DB the lesser Leg the proportion is,

	aD		AD		Db		DB
As	3,027	:	86	: :	1,00	:	28,4 required.

1,00

3,027) 86,00 (28,41 d.

25460

12440

332

Unto D E	73
Add DB	28,41
The Sum is EB	101,41

Then in the second Triangle E A B.
There is Given,

EB the base 101,41 and } To find A E the Hypothenuse and the angle AEB.
AB the perpendicular 81,2

The first Operation to find AE the Hypothenuse by the square-Root.

The Leg E B is	101,4	The Leg A B	81,2
mult. in it self	101,4	mult. in it self	81,2
	4056		1624
	1014		812
	10140		6496
		squ. of AB is	6593,44
The Squ. of E B is	10281,96		
The Squ. of AB added	6593,44		

The Squ. of AE is 16874,40 (129,9 AE required.
1

22) 68
44

249) 2474
2241

2589) 23300
23301

The second Operation is (having the three sides) to find the angle AEB opposite to the lesser leg A B.

The Hypothenuse AE is	129,9
Half of the greater Leg EB is	50,7
The sum is	180,6
The lesser Leg AB is	81,2

As

As 180,6 : 81,2 : : 86 : 38,7 *d.*
86

4872
6496

180,6) 6983,2 (38,7 *d. fere* ABE required.

15652

12040

Answer { A E is 129,9 or 130 ; A E D 38,7*d. fere* or 38*d.* 41*m.* ; and DAE 32 *d.* } which was required.

The Fourth Case.

THE three Sides being Given, To find the angles.

First Example.

In the Triangle A D E. There is given.

A E 112 }
A D 78 } AED and A D E required. Fig.20.
D E 62 }

For the Resolution of this Case, reduce the Oblique plain Triangle into two Right angled Triangles by letting fall a perpendicular, which may either fall within or without the Triangle without limation, that side upon which the perpendicular is let fall is the base. Then to find the bases in both the right angled Triangles, first find the alternate base by this proportion, *viz.*

As the true base, is to the sum of the two sides; so is the difference of those sides, To the alternate base: When the perpendicular is let fall within, this alternate base is the difference of the bases, and when the perpendicular falls without it is the sum of the bases in the two right angled Triangles; So that having the true and alternate base, substract the lesser from the greater, and the perpendicular falls in the middle of the remainder, therefore half the remainder is the base in the lesser Triangle, and half the remainder added to the alternate base if its the least, or to the true base if its the least, gives the base in the greater Triangle, and when the perpendicular falls within, the true base is the greatest; when without the Triangle, the alternate base is the greatest.

Thus having found the base in each Triangle, there is the base and hypothenuse in either known. To find the perpendicular (which is common to both) by the Square Root, which done, then is the three sides in each known to find there angles.

For better illustration of what is here premised, I shall let fall the perpendicular within in this first Example, and without the Triangle in the second. So that in this first Example the perpendicular D B is let fall from D the greatest angle, upon A E the greatest side, and describing the Semi-circle Z E X, with the extent DE the lesser side, upon D as a Center, then is A Z the sum, and AX the difference of the two sides AD and DE, also AE the true base and A N the alternate base, being here the difference between the bases A B and B E.

The first Operation is to find A N the alternate base.

AD is 78 } The two sides.
DE is 62 }

The sum is 140 equal A Z.

The differ. is 16 equal A X.

The

The Proportion,

As	AE	:	AZ	: :	AX	:	AN
	112	:	140	: :	16	:	20 the alternate base.
			16				
			840				
			140				

112) 2240 (20

8

From AE the true base	112
Substract AN the alternate base	20
The Remainder is N E	92 half whereof.
Is the Base BE in the lesser Triangle	46
Unto which add AN the altern. base	20
The sum is the base AB	66 in the great. Triangle.

The second Operation to find DB the perpendicular.

The Hypothenuse	DE is 62	The Base BE is	46
Which square	62	which square	46
	124		276
	372		184
Squ. of the Hypoth. DE	3844	Squ. of BE	2116
Squ. of B E substr.	2116		
Rem. the squ. of DB	1728 (41.57 is DB required.		

16

81) 128

81

825) 4700

4125

8307) 57500

The

The Third Operation to find the angle BED in the right angled plain Triangle EBD.

The Hypothenufe DE is	62
Half of EB the greater leg is	23
The Sum is	85
DB the leffer leg is	41,57

As 85 : 41,57 : : 86 : 42,1 *d.* the angle BED.

```
      41,57
         86
     ------
      24942
     33256
     ------
85)  3575,02 (92.1 d. fere the Complement where-
     ------          of to 90 d. is 47,9 d. the an-
       175           gle EDB.
     ------
         50
```

The fourth Operation to find the angle at A in the right angled Triangle ABD.

The Hypothenufe AD	78
Half of AB the greater leg is	33
The Sum is	111
DB the leffer leg is	41,57

As

As 111 : 41,57 : : 86 : 32,2 *d.* the angle BAD.

```
         41,57
            86
        -------
         24942
        33256
        -------                                deg.
  111) 3575,02 (32,2 d.the Compl.whereof to 90
        -------                             ------
           245       is the angle ADB        57,8
        -------         add the angle ADB    47,9
           230                              ------
        -------      sum is the angle ADE   105,7
             8        which was required---------
        -------
```

The Second Example of the Fourth Case.

IN the Triangle A D E. Fig. 21.
There is Given

AE 105 }
AD 69 } DAE, and ADE required.
DE 58 }

In this Example, I have let fall the perpendicular A B from the angular point at A without the Triangle upon the side D E continued which is here the base.

Therefore upon A as a Center with the extent of A D the lesser side, describe the Semi-circle XNZ, then is EZ the sum, and EX the difference of the two sides AE and AD, also DE is the true base, and N E the alternate base, being now the sum of the bases BD (equal BN) and BE.

The

The first Operation is to find EN the alternate base.

The two Sides.

AE	105
AD	69
The sum is	174 equal to EZ
The difference is	36 equal to EX Then say,

	DE	EZ	EX	EN
As	58 :	174 : :	36 :	108 the alternate Base.

174
36
———
1044
522
———
58) 8264 (108
464
———
0

From EN	108 the alternate Base.
Substract ADE	58 the true base.
There Remains DN	50
Half whereof is DB	25 the base in the lesser Triangle.
Unto which added DE	58 the true base.
The sum is BE	83 the base in the greater Triangle.

The

The second Operation to find AB the perpendicular.

The Hypothenuse AD is	69	The leg DB is	25
Which square	69	Which square	25
	621		125
	414		50
The squ. of A D is	4761	The squ. DB is	625
The squ. of DB is	625		

The squ. of A B is 4136 (64,31 is A B required.

36

124) 536

496

1283) 4000

3849

151

The Third Operation to find the angle at A in the Triangle DAB.

The Hypothenuse AD is	69
Half of AB the greater Leg is	32
The Sum is	101
DB the lesser leg is	25

As 101 : 25 : : 86 : 21 *d.* 28 *m.* the angle BAD.

```
         86
       ----
        150
       200
       ----
101) 2150 (21,28 d. or 21 d. 17 m. the Comple-
   ----       ment of which to 90 d. is the angle
      130     ADB 68 d. 43 m. and the Comple-
   ----       ment of 68 d. 43 m. to 180 d. is
      290     111 d. 17 m. the angle ADE required.
   ----
      880
```

The fourth Operation to find the angle AEB in the Triangle E AB.

The Hypothenuse AE is	105
Half of EB the greater leg is	41,5
The sum is	146,5
AB the lesser leg is	64,3

As 146,5 : 64,3 : : 86 : 37,75 *d.* the angle AEB.

```
           86
         ----
          3858
         5144
         ----
146,5) 5529,8 (37,75 d. or 37 d. 45 m. the Comple-
       ----            ment of which to 90 d. is
        11348          52 d. 15 m. the angle B A E
       ----            from which substract 21 d.
         10930         17 m. the angle BAD, remain-
       ----            der is 30 d. 58 m. the angle
           6750        DAE which required.
```

Answer ADE is 111 *d.* 17 *m.* and } which was
DAE is 30 *d.* 58 *m.* } required.

Finis Coronat Opus.

Thus

Thus much for the Doctrine of oblique angled plain Triangles by Natural Arithmetick. Whereby all the usual Cases therein are resolved without Tables, and no further burdensome to the memory, then the remembrance of three or four lines which is inconsiderable.

And it may also be observed, That all the answers for the foregoing Examples, in the several Cases of oblique plain Triangles rarely differ three Centisms from their Logarithmical answers; yet notwithstanding if any shall be so critical (or rather prejudicially prepossest) as to cavil at what is here undertaken, alledging it tedious, or that the rise thereof was from *Snellius*, or Mr. *Collins*, and long since made publick by them. To which my answer is, I must need acknowledge my self very much beholden to those worthy Men, whose Works was a means that this was brought to light; yet neither of them has given any Rule whereby to resolve a right angled plain Triangle, having only one of the sides and the angles given without the aid of Tables. Therefore let such as cavil or raise objections as aforesaid, but frame a better or more expeditious way from *Snellius* his Rule and Demonstration, or by any other means of their own without Tables, and then I shall very submissively crave their pardon; otherwise I may expect their favourable Construction of what I have here published.

I shall forbear the application of plain Triangles, to any parts of the Mathematicks; presupposing the Reader to have made so far an Introduction in such Sciences as to do that himself, but if not, shall refer him to some Authors already extant; many whereof have so well performed it, that now it seems almost impossible to add any thing more that is material or useful. Yet how advantagious this method may be at Sea in case bothbooks and instruments be lost, and how convenient on shore in taking altitudes and distances, when the Tables of Natural or Artificial Sines, Tangents and Logarithms are wanting, I shall leave to the impartial Practitioner to Judge.

And least any into whose hand this may come, should be at a loss for want of a competent knowledge in Decimal Arithmetick. I have hereunto subjoyned a short but full Treatise thereof, together with the Extraction of the Square Root. So that the Reader may herein be fully provided and fitted with what is most necessary and pertinent for performance of the preceeding Work.

Deci-

Decimal Arithmetick.

DEcimal Arithmetick derives its name from the intent, or meaning of the word as implying the integer (whether it be Money, Weight, Measure, Time or Motion, *&c.*) to be divided into 10 equal parts, each of those parts sub-divided into 10 equal parts more, and each of those into 10 again, *&c. ad infinitum.* So that hereby the integer is divided into 10, 100, or 1000 *&c.* equal parts, which are the Denominators to the Decimal Fractions, & therefore they whether alone or joined to whole Numbers are seldom exprest, but only the Numerators, which are distinguished from the whole numbers by a Comma, thus (,) having always for their Denominators an unite with as many Cyphers annexed to it, as their are Decimal places in the Numerators, as in these being joyned to whole numbers, *viz.* 3,2 : 33,75 : 126,632 and such like, which are read thus, 3 integers, and 2 tenths, 33 integers and 75 hundred parts, 126 integers, and 632 thousand parts, *&c.* or in these being alone, *viz.* ,5 : ,25 : ,645, *&c.* are read thus, 5 tenths, 25 hundred parts, 645 thousand parts, *&c.* hence its apppatent that, The denominator

$$\text{of}\begin{cases} ,5 & \text{is} \quad 10 \\ ,25 & \text{is} \quad 100 \\ ,645 & \text{is} \quad 1000. \end{cases}$$

The like denominators have those joyned to whole numbers;

IN Decimal Fractions, the value or Denomination of every figure or Cypher decreases by a tenfold proportion, from the units place towards the right hand, as the whole numbers do increase towards the left hand, as in the following Table.

 6—hun-

Of an Unite or 1.

6	Hundreds of Thousands.
5	Tens of Thousands.
4	Thousands.
3	Hundreds.
2	Tens.
1	Units.
,2	Tenth parts.
3	Hundred parts.
4	Thousand parts.
5	Ten Thousand parts.
6	Hundred Thousand parts.

Therefore Cyphers annexed to a Decimal Fraction alters not his value. As thus

,50
,500
,5000

Each being equal to 5 tenths of an integer.

The like is to be understood of all others neglecting the cyphers as insignificant; but,

Cyphers prefixed to a Decimal Fraction decreases its value by a tenfold proportion:.

As thus		is		
	,5		5 Tenths	parts of an integer or 1.
	,05		5 Hundred	
	,005		5 Thousand	
	,0005		5 Ten Thousand	

Here the denominator of		is	
	,5		10
	,05		100
	,005		1000
	,0005		10000

Ad-

Addition and Substraction of Decimals.

IN Addition or Substraction of Decimals, (whether alone or joined to whole numbers) observe to place every figure (as well of the Decimals as the integers) under that of the like value or place, that is tens under tens, and hundreds under hundreds, *&c.* Then add or substract as in integers of one denomination, distinguishing from the Sum or Remainder, so many figures for Decimals as are the most Decimal places in any of the given Numbers.

Examples in Addition.

Add 45,2 : 67,3 : 192,7 : 20,0 : and 34,1 inches together. They are thus placed for Addition,

	Inches.
	45,2
	67,3
	192,7
	20,0
	34,1
Sum	359,3

Add 125,3 : 46,15 : 304,95 : 120,05 : 409,26 feet together.

	Feet.
	125,3
	46,15
	304,95
	120,06
	409,26
Sum	1005,72

Add together 135,5 : 59,24 : 224,08 : 146,3 : 425,025 and 29,125 degrees.

	Degrees.
	135,5
	59,24
	224,08
	146,3
	425,025
	29,125
Sum	1019,27

Examples in Subſtraction.

From 495,75 feet, Subſtract 368,425 feet. Thus placed.

	feet.
From	495,75
Subſtract	368,425
Remains	127,325

Subſtract 29,736 degrees, from 43,025 degrees.

	degrees.
From	43,025
Subſtract	29,736
Remains	13,287

The proof of Addition and Subſtraction of Decimals, is the ſame as of Addition and Subſtraction of whole numbers in vulgar Arithmetick.

Multi-

Multiplication of Decimals.

IN Multiplication of Decimals (whether the numbers be Decimals alone or mixt numbers, that is Decimals annexed to whole numbers) having placed the Multiplier under the Multiplicand, proceed therewith in every respect as Multiplication of whole numbers of one denomination; distinguishing from the product on the right hand thereof so many figures for Decimals, as are the number of Decimal places in both the given numbers.

In Multiplication of Decimals alone (that is when the Factors are each less then an unit) the product thence arising is always less then either of them.

Hence it often happens that after Multiplication is finished, there are not so many figures in the product, as there are Decimal places in both the Factors. Then in such cases prefix as many cyphers on the left hand of the product as will supply that defect; and this defect will always happen when either one or both of the Factors, or given numbers are below the first place of Decimals.

Examples in Multiplication.

Factors	81,3	580,34
	34,7	47,5
	5691	290170
	3252	406238
	2439	232136
Product	2821,11	27566,150

6,75	,25
,25	,13
3375	75
1350	25
1,6875	,0325

,0476	,07293
,0245	,00453
2380	21879
1904	36465
952	29172
,00116620	,0003303729

In these three last Examples there not arising so many figures in the products, as there are Decimals places in the two Factors, the places wanting are supplyed by prefixing cyphers, according to the foregoing Rule.

Note, That when a Decimal Fraction or mixt number is to be multiplyed by an Unit with cyphers annexed thereto, (as 10, 100, 1000, *&c.*) it is but removing the comma, so many places further towards the right hand in the Multiplicand, as there are cyphers annexed to the unit. Thus if ,7652 were to be multiplyed, by

10	The product will be	7,652	The like of all others.
100		76,52	
1000		765,2	
10000		7652	

The

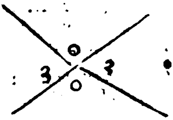
The Proof of Multiplication.

Multiplication may be proved thus, *viz.* First make a cross, then collect together all the significant figures in each Factor after the manner of Addition, and as often as it arises to 9 or more, cast away 9 and carry the Remainder to the next figure, and so proceed till all the figures are finished; setting the two last Remainders, one on the right, and the other on the left hand side of the cross. Multiply those remainders together, and from their product cast away all the 9's, and place that Remainder over the cross. Lastly cast away all the 9's (after the same manner) out of the product made by the two Factors, and if this Remainder be equal to that over the cross, the operation is true;

Example of the last preceding.

Factors $\left\{ \begin{array}{l} ,07293 \\ ,00453 \end{array} \right.$

Product ,0003303729 $\Big\}$ Proof.



Division of Decimals.

IN Division of Decimals (whether they be Decimals alone, or mixt numbers) having placed the dividend and divisor, according to that manner of division which you most affect in vulgar Arithmetick. The operation is in every respect the same as in whole numbers, distinguishing from the quotient towards the right hand, so many figures for Decimals, as that the number of Decimal places in the divisor and quotient, may be equal to those in the dividend. Or (If you please) take this Rule equivalent and agreeing with the former, in distinguishing the true value of the quotient *viz.*

Ob-

bserve under what place of the dividend the unites place
1e divisor will be found. And of the same place or de-
iinatiou is the first figure in the quotient, whether whole
iber or Decimal;
ere note that if neither the dividend nor divisor con-
of any Decimals, yet if there be a Remainder after the
sion is finished, it is necessary to continue the work by
exing cyphers to the dividend, till you have as many
es of Decimals in the quotient as you desire, or as the
iracy of your case requires: and in this Case how many
hers as is annexed to the dividend, so many Decimal
es there will be in the quotient.
lsoif the divisor is greater or if it consist of more Deci-
places then the dividend, annex to the dividend a com-
ent number of cyphers instead of Decimals to the end
you may have as many Decimals in the quotient as may
ccurate enough for your purpose, regard had to the for-
Rules, for right valuing of the quotient.
'o those that are unacquainted with division, I shall re-
imend the method following as the most commodious,
ards which, (because division is more difficult then any
he former species) I shall give such directions as (I pre-
e) may render the work intelligible to any reasonable
icity.

Examples in Division.

et it be required, To divide 736,534 by 32,5. First,
e the divisor 32,5 on the left hand of the dividend
,534 thus

32,5) 736,534 (2

86

hen see unto what figure in the dividend, the last figure
of the divisor will extend, (if it were orderly placed
er the same) which here is 6, over which place a point.

That

That done see how often the divisor (32,5) is contained in (736) the three first figures of the dividend, that is how often 3 in 7; the answer is 2 which place in the quotient, and thereby multiply the divisor in your mind, substracting the multiple of every figure orderly out of the dividend beginning at 6, thus 2 times 5 is 10, 0 from 6 there remains 6, then 2 times 2 is 4, and 1 I carry is 5, 5 from 3 I cannot, but borrowing 1 from 4 (the next figure) makes the 3, 13; then 5 from 13 there remains 8, again 2 times 3 is 6 and 1 I carry is 7, then 7 from 7 there rests nothing: now to this remainder (86) I bring down the next figure in the dividend which is 5, and place it on the right hand thereof for a new dividual and the work will stand thus,

```
32,5) 736,534 (2
      -------
       865
```

Then proceed in the work, and see how often the divisor (32,5) is contained in this new dividual (865) as before. I find 2 times, which place in the quotient and multiply the divisor thereby, and substract the product orderly as you go from the dividual as before, and subscribe the remainder; to which bring down the next figure 3 in the dividend, and so continue the work till all the figures in the dividend be finished, placing points over every figure thereof as it is brought down to prevent mistake; which done if there be any remainder you may continue the division, by annexing cyphers thereto, till you have as many Decimals in the quotient as may answer your intended accuracy. In this Case there is a Remainder: and the work being continued by annexing two cyphers, there will (according to the foregoing Rules) be four places of Decimals in the quotient, then the work stands thus,

```
32,5) 736,534 (22,6625
      -------
       865
      -----
       2153
      -----
       2034
      -----
        840
      -----
       1900
      -----
Remain  275
      -----
```

More Examples in Division.

Let it be required to divide 580,34 by 47,5. In this Example the work is continued, by annexing 3 cyphers to the Remainder, thereby there arising four places of Decimals in the quotient, *&c.*

```
47,5) 580,34 (12,2176
      ------
       1053
      ------
       1034
      ------
        840
      ------
       3650
      ------
       3250
      ------
        400 Remain.
```

Let be required to divide 32, by 51,2. Here the divisor is greater then the dividend but by annexing four cyphers to perform the division. There will (according to the former Rules) arise three Decimal places in the quotient, and therefore all the figures thereof are Decimals.

```
51,2) 32,00 (,625
      -----
       1280
      -----
       2560
      -----
         00
```

If (in this Example) the divisor had been all integers (*viz.* 512) the quotient would have been ,0625 and if 5120, the quotient would have been ,00625 which for better illustration I shall add two more Examples.

Divide

Divide ,07864, by 25.

```
25) ,07864 (,00314
      36
      114
       14
```

Divide ,00116620 by ,0476.

```
,0476) ,00116620 (,0245
          2142
           2380
             00
```

Note, That when any number (either Decimal or mixt) is given to be divided by an unit with cyphers annexed thereto (as 10, 100, 1000, *&c.*) it is only removing the comma in the dividend. So many places farther towards the left hand, as there are cyphers annexed to the unit, prefixing cyphers to the dividend, to supply the vacant places (if need be.) Thus if 7562 were to be divided

By	10 100 1000 10000 100000	The quotient is	756,2 75,62 7,562 ,7562 ,07562	The like of all others.

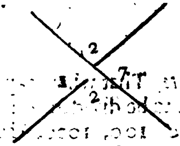
The

The Proof of Division.

DIvision may alſo be proved by collecting together all the ſignificant figures of the dividend, caſting away all the 9's as often as it ariſes thereto, (as has been already ſhown in Multiplication) and place the laſt remainder over the croſs, likewiſe caſt away all the 9's out of the diviſor and quotient, ſetting the laſt remainder of each, one on the left and the other on the right hand ſide of the croſs. Then multiply thoſe two remainders together, adding to their product the remainder after diviſion is ended (if there were any) and from that ſum caſt away all the 9's, and if this laſt remainder be equal to that over the croſs, the operation is true.

Example of the ſecond preceeding, wherein

The dividend is	580,34
Diviſor	47:5
Quotient	12,1276
Remainder	400



But the moſt certain proof of Diviſion is by Multiplication, and likewiſe of multiplication by diviſion: For in multiplication the product being divided by either of the Factors, the quotient is the other.

Alſo in Diviſion, the diviſor and quotient multiplied together adding thereto the remainder after diviſion (if any be) gives the dividend: The laſt example but one in multiplication is the laſt in diviſion, and ſerves to verifie theſe two laſt Rules. Alſo Diviſion may be proved by Diviſion, for if the dividend be divided by the quotient it will quote the firſt diviſor.

Exam-

Example.

,25) ,0325 (,13 ,13) ,0325 (,25

75	65
0	0

The like also holds good in multiplication, if first one of the Factors be made the multiplier, and then the other.

Example.

,25	,13
,13	,25
75	65
25	26
,0325	,0325

Reduction of Decimals.

TO Reduce a Vulgar Fraction into a Decimal equivalent thereto.

This Proportion holds. As the denominator of any Vulgar Fraction, Is to his numerator. So is 1,00, *&c.* the denominator of the Decimal Fraction, To his corresponding numerator required. And the Converse hereof serves to Reduce Decimals into Vulgar Fractions.

Hence it is, that if to the numerator of any Vulgar Fraction you annex as many cyphers as you intend, its equivalent Decimal shall consist of places, and divide it by its denominator the quotient is the Decimal required, which if nothing re-

remain is precisely equal in value to the Vulgar Fraction pre-propofed. But if there be a remainder after division you may (if you please) make a nearer approach by annexing more cyphers and continuing the work, so will the Decimal be infinitely near equal to the Vulgar Fraction given: Regard had to the former Rules in division, for right valuing of (or distinguishing the Decimals in) the quotient.

Note, That the aliquot or even parts of an integer may be reduced into Decimals, exactly equal thereto without a Remainder: But the odd parts of an integer cannot, for there will always be a Remainder. However if you carry on the Fraction to four or five places of Decimals, it will be exact enough in most Cases.

Examples.

Reduce $\frac{3}{4}$ into its equivalent Decimal, and it is ,75 thus,

4) 3,0 (,75
20

$\frac{1}{4}$ being Reduced is ,25 thus, 4) 1,0 (,25
20

$\frac{1}{8}$ Reduced is ,125 thus 8) 1,0 (,125
20
40

$\frac{4}{9}$ Be-

$\frac{4}{9}$ Being Reduced makes ,4444 &c. nearly equal to $\frac{4}{9}$.

9) 4,0 (,4444 &c. *ad infinitum.*

40

40

40

Reduce $\frac{9}{11}$ into a decimal and it is ,8182 *fere* thus,

11) 9,0 (,8182

20

90

20

Reduce $\frac{11}{941}$ into a Decimal and it is ,011689, and may be continued further.

941) 11,00 (,011689..

1590

6490

8440

912

Also $\frac{23}{5642}$ being Reduced is ,004076 &c.

And likewise $\frac{14}{64837}$ being Reduced is ,000216, *fere*.

To

To Reduce the known parts of an Interger (as Money, Weight, Measure, Time, Motion, *&c.*) *into Decimals.*

First, Reduce the given parts into a Vulgar Fraction whose denominator is the number of those known parts contained in the integer, and the given parts is the numerator thereof; but if the given parts be in several denominations, Reduce them into the least mentioned for the numerator, whose corresponding denominator, is the number of the same known parts, continued in the integer, which done Reduce such Vulgar Fractions into Decimals, by preceeding Rules, continuing the work to what accuracy you please, for the further you proceed the nearer is the approach.

Examples.

Reduce 5 inches in Decimals of a foot. First according to the last preceeding Rule, 5 inches being exprest after the manner of a Vulgar Fraction is $\frac{5}{12}$ of a foot, and the Decimal corresponding by Reduction is ,4166, *&c.* or ,41667 *&c.* Thus,

12) 5,0 (,4166, *&c.*

20

80

80

80

Reduce 7 inches $\frac{3}{4}$ into the Decimal of a foot.

$7\frac{3}{4}$ Reduce into quarters of an Inch is 31

$7\frac{3}{4}$
4
31

and

$$\begin{array}{r} 12 \\ 4 \\ \hline 48 \end{array}$$

and 12 inches is 48 quarters, of an inch

Therefore the Vulgar Fraction is $\frac{31}{48}$ of a foot, which Reduced gives this Decimal, ,64583, &c.

Reduce 11 s. 6 d. into the decimal of a pound *sterling*.

$$\begin{array}{r} 11-6 \\ 12 \\ \hline \end{array} \qquad \begin{array}{r} 20 \\ 12 \\ \hline \end{array}$$

11 s. 6 d. reduc. into d. is 138 pence, and 1 l. reduc. is 240 d. therefore the Vulgar Fraction is $\frac{138}{240}$ or $\frac{23}{40}$ of 1 l. whose, equivalent decimal is ,575.

Note, That when a Vulgar Fraction is proposed to be Reduced into a Decimal, first abreviate or reduce the Vulgar Fraction into its least terms, by dividing both the numerator and denominator thereof, by any number that will evenly divide them; as $\frac{138}{240}$ both the numerator and denominator being divided by 6, gives $\frac{23}{40}$.

If 6 d. $\frac{3}{4}$ be Reduced into a decimal of 1 l. *sterling*, the Vulgar Fraction is $\frac{27}{960}$ or $\frac{9}{320}$ (being divided by 3,) and its equivalent decimal is ,028125.

Reduce 1 *quarter* 21 *l.* into Decimals of a C. weight, 1 *qu.* 21 *l.* Reduced into pounds is 49, and the pounds in 1 C. is 112, the Vulgar Fraction is $\frac{49}{112}$ or $\frac{5}{7}$ of a C. weight, (being divided by) whose equivalent decimal is ,4375.

Reduce 16 *l.* 7 *ounces* into the decimal of a C. weight:

multiply 16 7	112	Hence the vulgar
by 16	16	Fraction is $\frac{263}{1792}$,
		and the decimal
product is 263 *ounces*.	1792 *ounces*	corresponding is ,14673, &c.

Reduce 42 minutes into decimals of a degree, the vulgar fraction is $\frac{42}{60}$ and its equivalent decimal is ,7.

Reduce 23 minutes into the decimals of a degree the vulgar fraction is $\frac{23}{60}$ which gives this decimal 3833, &c.

The like is to be underſtood in Reducing the known parts of any other integer into Decimals, (as the Yard, Pole, liquid or dry meaſure, and ſuch like) whereby Tables may be calculated to Reduce the common known parts of any Integer into decimals, and the contrary by inſpection, as are in divers Authors but here (for brevity ſake) omitted, ſave only a Table ſerving to convert Sexagenary Fractions or minutes into decimals; and the contrary whoſe uſe being ſo eaſie that both explanation, and more examples are needleſs; which Table becauſe it is ſubſervient to the preceding Treatiſe of Arithmetical Trigonometry, is ordered that it may lye open to view at the reading any part thereof.

To Reduce a Decimal Fraction into the Common known parts of an Integer: Otherwiſe called Valuation of Decimals.

The Rule is,

Multiply the given Decimal (according to the preceding Rules for Multiplication) by the number of known parts of the next lower denomination contained in the Integer, unto which the given decimal hath relation, the product is the value of the ſaid decimal in the ſame denomination, and the ſeparated decimals (if any be) are decimals of an Integer of the like denomination with the multiplicator, the value whereof may (in like manner) be found in known parts of the next inferiour denomination, and ſo proceeding to the leaſt known parts of an integer, the whole numbers in the reſpective products ſhews the value of the decimal propoſed in the ſeveral known parts of that integer, whereof the given Decimal is ſome part or parts, as in the following

Exam-

Examples.

Reduce ,41667 of a Foot into inches

According to the preceding Rule multiply by 12 (the inches in a foot) and the Answer is 5 inches.

,41667
12
inch. 5 | ,00004

How many inches and quarters of an Inch is ,64583 of a Foot: The Answer is 7 inches and 2,999 quarters, or 7 inches 3 quarters.

,64583
12
inch. 7 | ,74996
4
quart. 2 | ,99984

Reduce ,575 of a pound Sterling into Shillings and Pence: The Answer is 11 s. 6 d.

,575
20
shill. 11 | ,500
12
pence 6 | ,000

What is the value of ,877083 *l.* in Shillings, pence and farthings, *&c.*

The Answer is 17 s. 6 d. 1 qu. ,999 or 17 s. 6 d. $\frac{1}{2}$ *fart.*

,877083
20
Shill. 17 | ,541660
12
Pence 6 | ,499920
4
Farth. 1 | ,999680

What

Whats the value of ,028125 *l. sterling?*

Answer 6 d. ¾

,028125
20
Shill. ,562500
12
Pence 6 | ,750000
4
farth. 3 | ,00

Reduce ,4375 of a C. weight into quarters and pounds.

Answer 1 quart. 21 *l.*

,43[illegible]
quart. 1 | ,7500[illegible]
28
60000
15000
l. 21 | ,0000

How many quarters, pounds and ounces is ,146763 of a C. weight.

Answer 16 *l.* 6 *oun.* ,999296 or 16 *l.* 7 *oun. prope.*

,146763
4
quart. ,587052
4696416
1174104
pounds 16 | ,437456
16
2624736
437456
ounc. 6 | ,999296

How

How many minutes is, 7 of a degree,	,7 60
Answer 42 minutes.	42,0
Reduce ,3833 of a degree into minutes.	,3833 60
Answer 22,998 or 23 min. *prope*,	22,9980

[illegible] to Discover the value of any Decimal [illegible] of a pound sterling, viz.

HAlf the number of Shillings being Tenths of a pound. Therefore the figure in the first place (*viz.* the place of Tens) being doubled gives the number of Shillings, unto which if the figure in the second place be 5 or above, add 1 shilling, then the figure in the second place if less then 5, or its excess if greater being esteemed as so many Tens, and the figure in the third place as so many units, and both accounted one intire number which lessened by 1 gives the number of farthings which together with the Shillings before found, are the value of the Decimal proposed, and the remaining figures (if any be) are the Decimals of a farthing.

As in the preceding Example of Reducing ,575 of a pound sterling into shillings and pence. Thus two times 5 is 10 s. unto which add 1 s. (because the next figure 7 exceeds 5 by 2,) and it makes 11 shillings, then there remains 25, which less by ,1 is 24 the farthings, and 24 farthings is 6 pence: so that the value is 11 s. 6 d. as before found; ——The like is to be underst[illegible]d of any other Decimal of a pound.

From what hath [illegible] premised of Reducing of vulgar Fractions into Decimals, after the same manner when any number is given to be divided by another (whether the giv-

en Divisor be a whole number or vulgar Fraction.) You may find a Decimal multiplicator which shall effect the same as the Divisor given;

To find which this Proportion holds, *viz*.

As the Divisor given : Is to 1 : : So is 1 : To the multiplicator required, thus if the divisor is greater then an unit the multiplicator is less, in the like Geometrical progression and the contrary, As if the divisor were 2 then (according to the preceding proportion) the multiplicator is $\frac{1}{2}$ or ,5 (its equivalent Decimal) it being the same to multiply any number by $\frac{1}{2}$ as to divide it by 2 : And hence it follow[illegible] 1 is always a Geometrical mean between the divisor a[illegible] multiplicator.

First Example, The Divisor being a whole Number.

Suppose it were required to divide 900 by 25, the quotient will be 36, Now to find the multiplicator, say

As 25 : 1 : : 1 : $\frac{1}{25}$ the multiplicator required, it being the same to multiply by $\frac{1}{25}$ as to divide by 25, and ,04 the Decimal of $\frac{1}{25}$ is the *Decimal* multiplicator sought.

thus	900
multiplied by	,04
the product is	36,00 which was required.

Second Example; The Divisor being a vulgar Fraction.

Suppose it were required to divide 345 by $\frac{5}{8}$ (according to division in vulgar Fractions) multiply the dividend 345 by 8 and divide that product (2760) by 5 the result is 552 the quotient required : or 345 divided by ,625 the decimal of $\frac{5}{8}$ quotes 552, but to find the multiplicator it holds. As $\frac{5}{8}$: 1 : : 1 : $\frac{8}{5}$ the multiplicator required, and therefore

fore it is the fame to multiply by $\frac{8}{5}$ as to divide by $\frac{5}{8}$, and 1,6 the equivalent mixt decimal of $\frac{8}{5}$ is the decimal multiplicator fought. Or having the decimal divifor ,625 fay : As ,625 : 1 : : 1 : 1,6 the multiplicator as before.

Therefore	345
multiplied by	1,6
produces	552,0 which was required.

Hence it is evident that the refult of the preceding Rule for finding multiplicators is, That if the given divifor be a whole number, make it a denominator of a vulgar fraction and 1 the numerator thereof, which vulgar fraction, fhall be a multiplicator (agreeable to the preceding proportion) whofe equivalent Decimal fhall be a decimal multiplicator, and effect the fame as the divifor given ;

But if the given divifor be a vulgar Fraction (whether proper or improper) change the numerator and denominator into each others place, fo fhall there arife another vulgar fraction for a multiplicator (correfpondent with the preceding Analogy) whofe equivolent decimal fhall be the Decimal multiplicator required.

More Examples.

Let it be required to divide 13375 by 125, the quotient is ,107, the multiplicator in terms of a vulgar Fraction, will be $\frac{1}{125}$ and the equivalent decimal of $\frac{1}{125}$ is ,008 the Decimal multiplicator required.

thus	13375
multiplied by	,008
the product is	107,000 as before.

Alfo 86235 divided by 123 quotes 701,09. Therefore multiply 86235 by ,00113 (the decimal of $\frac{1}{123}$) the product is 701,09, &c.

 And

And suppose it were required to divide 18662 by $\frac{8}{25}$. The quotient is $\frac{466550}{8}$ or 58318,75 see the work after the manner of vulgar Fractions.

$\frac{8}{25}$) 18662
25

93310
37324

8) 466550(58318,75

Or, If 18662 be divided by ,32 (the equivalent decimal of $\frac{8}{25}$) the quotient is 58318,75 as before.

,32) 18662 (58318,75

266

102

60

280

240

160

But to find the decimal multiplicator,

Place the numerator and denominator interchangeably, the vulgar Fraction thence resulting is $\frac{25}{8}$ whose equivalent decimal is 3,125. The mixt decimal multiplicator required.

multiplicand 18662
multiplier 3,125

93310
37324
18662
55986

The product is 58318,750 equal the quotient before found.

If

If the divisor given be a mixt Number Reduce it into an improper fraction, as If 6984 were to be divided by $1\frac{1}{3}$ the divisor being Reduced into an improper fraction is $\frac{4}{3}$ and the quotient thereby is $\frac{20952}{4}$ or 5238 : also 1,3333 &c. (the decimal of $\frac{4}{3}$) is the mixt decimal divisor. likewise the Numerator and denominator interchangeably placed the fraction resulting is $\frac{3}{4}$ the equivalent decimal whereof is ,75 the Decimal multiplicator which was required.

multiplicand 6984
multiplicator ,75

34920
48888

The product is 5238,00 equal the quotient before (found.

What hath been already said concerning finding multiplicators instead of divisors, Holds good likewise in all such cases wherein there are two given numbers or terms expressing a fixt reason or proportion between any two quantities or magnitudes.

By proportion is understood the habitude or relation of two numbers (or quantities of the same kind) one to the other, and is found by dividing the antecedent or first term by the consequent or second term, and therefore they are Exprest in form of a fraction ; As if the proportion be of 5 to 7: Set thus $\frac{5}{7}$: if of 7 to 5 thus $\frac{7}{5}$;

The sum of what is here intended is; When there is two given terms expressing the proportion that some known Number hath to another unknown or required : place the two terms of proportion fractional wise making the Antecedent the Numerator, and Consequent the denominator, that vulgar fraction may be accounted as a divisor. But if the two terms be interchangeably placed, that is the consequent for the numerator, and the antecedent for the denominator, that vulgar fraction serves as a multiplicator, and their respective Decimals, shall be a decimal Factor and Divisor, which by either a single multiplication or division shall effect the

the same; as by multiplication and division of the two given terms.

First Example

The proportion of the diameter of a Circle to its circumference is, As 113 to 355, or $\frac{113}{355}$ and the contrary, the circumference to its diameter is, As 355 to 113: or $\frac{355}{113}$. Therefore having the diameter to find the circumference, ,3183 (the decimal of $\frac{113}{355}$) is a fixt divisor, and 3,1416 (the decimal of $\frac{355}{113}$) a fixt multiplicator: But if the circumference is known and the diameter required; then is 3,1416 a fixt divisor, and ,3183 a fixt Factor: also 3,1416 is the circumference of a circle whose Diameter is 1, and ,3183 the diameter of a circle whose circumference is 1. Thus if the diameter of a circle be 28,25, the circumference will be found to be 88,75, *&c.*

```
      3,1416          ,3183) 28,250 (88,75 &c,
       28,25                 ------
      ------                  27860
      157080                 ------
      62832                   23960
     251328                  ------
     62832                    16790
    ---------                ------
    88,750200                 875, &c.
```

Likewise of the circumference of a circle be given 88,75 the diameter (in like manner) will be found to be 28,25;

```
      88,75          3,1416) 88,750 (28,25
      ,3183                  ------
     ------                  259180
      26625                  ------
     71000                    78520
     8875                    ------
    26625                     15698
   ---------
   28,249125
```

Ex.

Example Second.

The ſquare of the diameter of a Circle is in proportion to the Area (or content) thereof. As 14 is to 11: or $\frac{14}{11}$ and contrarily the Area is to the ſquare of the diameter. As 11 to 14 or $\frac{11}{14}$. Therefore having the diameter to find the Area by this proportion; The Factor is ,7857 (the decimal of $\frac{11}{14}$) and the diviſor is 1,2727, (the decimal of $\frac{14}{11}$), by which the ſquare of the diameter being multiplyed or divided, there will reſult the Area; but if the Area is given to find the diameter by this proportion, then is 1,2727 the Factor, and ,7857 the diviſor, whereby multiplying or dividing the Area will give the ſquare of the diameter.

Note, that tho the proportion of 11 to 14 or 14 to 11 be uſed by divers Authors, in the two propoſitions aforeſaid, yet they are not accounted exact enough where a queſtion requires preciſeneſs. So that inſtead of ,7857 uſe ,7854, which is the Area of a circle whoſe diameter is 1 and inſtead of 1,2727 uſe 1,27324 (found by dividing an unit with cyphers by ,7854) which is alſo the ſquare of the diameter of a circle, whoſe Area 1; Thus the diameter of a circle being 28,25 the Area or ſuperficial content will be 626,7983 *fere.*

The ſquare of the diameter	798,0625
multiplied by	,7854
	31922500
	39903125
	63845000
	55864375
The Area is	626,79828750 required.

Or

Or by dividing 798,0625, the square of the diameter by 1,27324 the fixt divisor:

1,27324) 798,0625 (626,797 fere the Area.

341185

865370

1014260

1229920

840040

In like manner the Area being given 626,798, &c. The square of the diameter will be found 798,0625, by dividing the Area by ,7854. Or which is the same, by multiplying the Area by 1,27324: and the square Root of 798,0625 is 28,25 (how to extract the square Root shall be hereafter shewed.)

Example Third.

The Cube of the diameter of a Globe or Sphere is in proportion to the solidity (or solid content thereof) As 21 is to 11: or $\frac{21}{11}$.

Therefore knowing the diameter, to find the solidity thereof according to this fixt Proportion.

The fixt $\left\{ \begin{array}{l} \text{Factor is ,5238 the decimal arising from } \frac{11}{21}. \\ \text{Divisor is 1,9091 the decimal arising from } \frac{21}{11}. \end{array} \right.$

Whereby multiply or divide the cube of the given diameter, and the quotient or product will give the solid content of the Globe or Sphere required.

But if the Solidity be given, to find the Diameter, then the proportion holds. As 11 to 21, so is the given Solidity of the

the Globe, to the Cube of its Diameter. So in this Case, ,5238 is a fixt Divisor and 1,9091 is a fixt Factor, by which Multiplying or Dividing the given Solidity, the result is the Cube of the Spheres Diameter required, but these Numbers, *viz.* 11 to 14 or 14 to 11 being not admitted as precise enough, therefore the Factor and Divisor thence resulting, is not to be accepted, where great exactness is required; so that instead of ,5238. use ,5236. (the Solidity of a Globe whose Diameter is 1) and instead of 1,9091, take 1,909855, (the Cube of the Diameter of a Globe whose Solidity is 1) found by dividing an Unit with Cyphers by ,5236: thus the Diameter of a Globe being 14, the Solid content will be found to be 1436,76, &c.

The Cube of 14 the Diam. is	2744
Multiplied by	,5236
	16464
	8232
	5488
	13720
The Solidity is	1436,7584

Or dividing 2744 the Cube of the Diameter by 1,909855 the fixed Divisor.

1,909855) 2744,000 (1436,7584 fere the Solidity.

8341450

7020300

12907350

14482200

11132150

15828750

549910

Likewise

Likewise had the Solidity of the Globe been proposed to be 1436,7584. the Cube of its Diameter will be found to be 2744 by dividing the solid content by ,5236, or multiplying by 1,909855.

More Examples of this nature might be enumerated, but because these things more properly relate to Geometry than Arithmetick, I shall refer the Reader to such Authors as treat of Mensuration and Gauging, and let these suffice to shew the advantage of Decimal Arithmetick in finding Multiplicators and Divisors, so shall proceed further to illustrate, the utility of Decimal Arithmetick in Rules of Plural proportion, *viz* such wherein the *Rule* of *Three* is several times repeated. And first of

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The Golden Rule or Rule of Three.

THe *Rule* of *Three* in Decimals may be performed in every respect, as in whole Numbers, of which its presupposed the Reader hath competent knowledge, or at least he ought to have before he enter upon Decimal Arithmetick.

Note, That when any of the three given terms in the question proposed, consists of Fractional parts, or have Fractions joyned thereto, reduce such Fractions or Fractional parts into Decimals, and then proceed to Operation,

Example.

C. l. s. d. C.
If $1\frac{1}{4}$ cost 2 16 04 what will $14\frac{1}{2}$ cost?

The Fractional parts being turn'd into Decimals, and the Question stated, stands thus,

If

	C.		l.		C.	
If	1,75	:	2,8125	:	14,5	:

```
          2,8125
            14,5
         -------
          140625
         112500
         28125
         -------
          .....       l.
1,75 ) 40,78125 (23,3036 fere
       578                20
       ----             -------
        531   shill. 6,0720
        ----             12
        625           -------
        ----       pen.,8640
        100               4
                      -------
                  farth. 3,3560
```

Here multiplying the second and third Numbers together, and dividing that product by the first, after the manner of vulgar Arithmetick. The quotient is the answer, *viz.*

l.		l.	s.	d.			C.	
23,3036	or	23	06	$00\frac{1}{4}$	and so much	will	$14\frac{1}{2}$	cost.

Also in the Operation of the *Rule of Three* in Decimals, you may divide the second Number by the first, and multiply the quotient by the third, and the product thereof answers the question; or (which will give the same result.) Divide the third number by the first and multiply the quotient, by the second, and by one of these two varieties, you may in some cases abreviate the work more than by the common way: Thus the preceding question wrought accordingly.

First, By dividing the third number by the first, &c.

C.	l.	C.
1,75 :	2,8125 : :	14,5 :

```
1,75) 145,0 (8,28571
      -----      2,8125
       500      -------
      -----     4142855
       1500     1657142
      -----     828571
       1000    6628568
      -----   1657142
       1250   --------------
      -----   23,303559375 l. The product
        25                (the answ. as before.
```

2dly. By dividing the second number by the first, &c.

```
1,75) 2,8125        (1,607143
      ------              14,5
       1062        -----------
      ------          8035715
       1250          6428572
      ------        1607143
        250        -----------
      ------        23,3035735 l.
        750
      ------
         50
```

Thus may any question in the *Rule of Three* be performed several ways, the practice whereof I leave to the Ingenious Reader, only add one more *Example*.

oz.	l.	s.	d.	l.	oz.

If 4 of Gold be worth 13 14 06 what are 5 9 worth, the Fractional parts being turn'd into Decimals, (the pound weight being the integer) and the question stated stands thus,

If

	l.		l.		l.
If	,333333	:	13,725	::	5,75

$$\begin{array}{r} 13{,}725 \\ 5{,}75 \\ \hline 68625 \\ 96075 \\ 68625 \\ \hline \end{array}$$

,333333) 78,91875 (236,7564 l.

1225215	20
2352160	shill. 15,1280
2521620	12
1882890	d. 1,5360
1622500	4
162252	farth. 2,144

Answer, the 9 oun. of Gold will cost 236 l. 15 s. $1\frac{1}{2}$ d.

Note, That instead of dividing by ,333 *&c.* (the decimal of $\frac{4}{12}$ or $\frac{1}{3}$) you may multiply by the Factor 3, it being the same (as hath been already shown) to multiply by 3 as to divide by $\frac{1}{3}$.

$$\begin{array}{r} 78{,}91875 \\ 3 \\ \hline 236{,}75625 \end{array}$$

Hence its evident that by finding of a Factor when a Divisor is given; or a Divisor having a Factor given, you may often abbreviate the work in the Rule of Three.

Also in working the preceding question according to the two last varieties, (mentioned in the first Example) instead of dividing either the second or third numbers by ,3333 *&c.* you may multiply by 3 its corresponding Factor which will much more facilitate the work.

l. l. l.
If ,33333 &c. (or $\frac{1}{3}$) : : 13,725 : : 5,75 :
Factor 3

41,175
5,75

205875
288225
205875

The product is 236,75625 as before.

Or if the second Number be divided by the first &c. the work will stand thus ;

,333333) 13,72500 (41,175
5,75

391680
205875
288225
583470
205875

2501370
236,75625

168939

Because I would use as much brevity as possible in order that other things (of good use) intended for this Treatise may come in, these two Examples in the Rule of Three may suffice, which if well regarded, the intelligent Reader may be able to perform the like by any other.

It may also be observed, That these two last varieties in the Rule of Three, especially that, where the number which is usually made the second term, is divided by the first, serves as a necessary allusion to contract the several tedious operations in the Rules of Fellowship and Alligation altrenate. Wherein there are required so many several proportions

portions or statings of the Rule of 3, as there are particular Proportional parts to be found, all whereof having the same Common factor & Divisor may therefore more expeditiously be performed in decimals by the following Rule. *Viz.* Divide the common factor or that number which is usually made the 2d term in the Rule of Three, by that which is always made the first or common Divisor. And multiply that quotient severally into the respective third Numbers. The particular products are, the proportional parts or numbers required.

But here it will be necessary if you use decimals (which is most convenient) to continue the Division (as long as there is a Remainder) till you have six places of decimals in the quotient. So shall the amount of the several parts, (in the proof of the work) not want $\frac{1}{1000}$ part of an unit of being equal to the second number or dividend.

First Example in Fellowship without Time.

THree Partners A B and C make a Stock of 625 l.

whereof { A put in 254 l. — B ——— 172 — C ——— 199 } Their Gain being 295 l. I Demand each Mans proportionable part thereof?

First, The Resolution hereof according to the Customary method is as follows;

The first Operation for A's part of the Gain.

If 625 *l.* : 295 *l.* : 254 *l.*

```
  254
 ------
 1180
1475
590
 ------
 74930
 ------
```

625) 74930 (119,888 is A's part of the Gain.
20
1243
shill. 17,760
6180
12
5550
d. 9,120
4
5500
f. ,480
500

The Second Operation for B's part of the Gain.

	l.	l.	l.
If	625 :	295 : :	172 :
		172	
		590	
		2065	
		295	

625) 50740 (81,184 is B's part of the Gain.
20
740
shill. 3,680
1150
12
5250
d. 8,160
4
2500
farth. 640

The

The Third Operation for C's part of the Gain.

```
 l.          l.        l.
If 625  :   295  :  :  199 :
            199
           -----
            2655
           2655
           295
           -----
                          l.
     625) 58705        (93,928 is C's part of the
         -------            20          (gain.
           2455        ---------
         --------  shill. 18,560
            5800              12
          -------      ---------
             1750      d. 6,720
           -------             4
              500      ---------
                    farth. 2,880
```

The Proof.

```
                  l.                  l.    s.   d.   f.
A's }         { 119,888 }          { 119 — 17 — 09 — 0,48
B's } Gain is {  81,184 } Or thus, {  81 — 03 — 08 — 0,64
C's }         {  93,928 }          {  93 — 18 — 06 — 2,88
               ---------            ----------------------
          Sum  295,000               295 — 00 — 00 — 0,00
               ---------            ----------------------
```

Secondly, The Resolution according to the new Method, is as follows, in which observe, That

In all the preceding Operations (or Statings) of the Rule of Three, 295 l. (the gain) is the common factor, and usually made

made the second number, and 625 (the whole Stock) is always the first or common Divisor. Therefore (according to the Rule,) Divide 295 by 625, &c,

```
625) 295,0    (,472
     -----   A  254
      4500     -----
     -----      1888
      1250     2360
               944
              -----
A's part is l. 119,888
              -----

   ,472
B   172
  -----
    944
   3304
   472
  -----
C's part is l. 81,184
  -----

   ,472
C   199
  -----
   4248
  4248
  472
  -----
C's part is l. 93,928
  -----
```

By this new method it is evident that the work is much more compendious then by the former: and the more the Partners are, the advantage of this method would be the more signally conspicuous, for one single Division sufficeth be the Partners, (or parts to be found) never so many.

A Se-

A Second Example of Fellowship without Time.

SIX Merchants, viz. A, B, C, D, E, and F make a Joint-Stock of 1944 *l.* wherewith (in a certain time by good management) they Gained 1215 *l.* each mans part of the Stock being, as follows.

	l.	*s.*			*l.*
A laid in	348	15	Or in Decimals	A	348,75
B	421	12		B	421,6
C	297	16		C	297,8
D	410	5		D	410,25
E	342	4		E	342,2
F	123	8		F	123,4
Sum	1944	00		Sum	1944,00

I demand each mans proportinal part of the Gain.

The Operations being performed according to the customary method, requires six proportions or statings of the Rule of Three, because there are six partners, which being tedious I shall only express the said proportions and their answers omitting the work.

First for A's Gain.

If 1944 *l.* : 1215 *l.* :: 348,75 *l.* Answer 217,96875 *l.*
Or 217 *l.* 19 *s.* 4½ *d.* is A's gain.

Secondly, For B's gain.

If 1944 *l.* : 1215 *l.* :: 421,6 *l.* Answer 263,5 *l.*
Or, 263 *l.* 10 *s.* is B's gain.

Thirdly, For C's gain.

	l.		l.		l.		l.
If	1944	:	1215	: :	297,8.	Answer	186,125

Or 186 *l.* 2 *s.* 6 *d.* C's is gain.

Fourthly, For D's gain.

	l.		l.		l.		l.
If	1944	:	1215	: :	410,25.	Answer	256,40625

Or 256 l. 8 s. $1\frac{1}{2}$ d. is D's gain.

Fifthly, For E's gain.

	l.		l.		l.		l.
If	1944	:	1215	: :	342,2.	Answer	213,875

Or 213 l. 17 s. 6 d. is E's gain.

Sixthly, For F's gain.

	l.		l.		l.		l.
If	1944	:	1215	: :	123,4.	Answer	77,125

Or 77 l. 2 s. 6 d. is F's gain.

The Operations being performed according to the new method, 1215 (the common factor and second number in all the preceding proportions) is the dividend, and 1944 (the first number) is the divisor.

1944) 1215,0 (,625 ;	A 348,75 l.	B 421,6 l.
	,625	,625
4860		
	174375	21080
9720	69750	8432
	209250	25296
	A's gain is *l.* 217,96875	B's 263,5000 *l.*

C 297,8 l.

C 297,8 *l.*	D 410,25 l.
,625	,625
14890	205125
5956	82050
17868	246150
C's gain is 186,1250 *l.*	D's gain is 256,40625 l.

E 342,2 *l.*	F 123,4 *l.*
,625	,625
17110	6170
6844	2468
20532	7404
E's gain is 213,8750 l.	F's gain is 77,1250 l.

The Proof.

	l.		*l.*	*s.*	*d.*
A's gain is	217,96875	Or, thus	217	19	$04\frac{1}{2}$
B's	263,5		263	10	00
C's	186,125		186	02	06
D's	256,40625		256	08	$01\frac{1}{2}$
E's	273,875		213	17	06
F's	77,125		77	02	06
Sum	1215,00000	Sum	1215	00	00

These two Examples in the Rule of Fellowship without time. (I presume) are sufficient as well to illustrate this speedy method in Decimals, as also, (if well observed.) The Reader may be thereby capable to Resolve any other of this nature by the same new method, as in the case of Bankrupts compounding with their Creditors and the like.

The Third Example is in the Rule of Fellowship with Time.

THree Merchants A, B and C enter into Partnership, *viz.*

A laid in 54 l. for 8 months B —— 85 —— 12 —— C —— 49 —— 4 ——	They gained 309 l. what is each mans part thereof?

In the Rule of Fellowship with Time, multiply each mans particular Stock and Time together, and the total of those products is the first number (or term) in the Rule of Three, the several products are the respective third numbers, and the whole loss or gain the second.

A's Stock is 54 l.
his Time is 8 mon.

product 432

B's Stock is 85 l.
his Time is 12 mon.

product 1020

C's Stock is 49 l.
his Time is 4

product 196

A's prod. 432
B's —— 1020
C's —— 196

Sum 1648

In the Resolution according to the customary method the several proportions are as follows, *viz.*

First, For A's gain.

If 1648 : 309 l. : : 432. Answer 81 l. A's gain.

Se-

Secondly, For B's gain.

If 1648 : 309 l. : : 1020. Answer 191,25 l,
Or, 191 l. 00 s. 6 d. B's gain.

Thirdly, For C's gain.

If 1648 : 309 l. : : 196. Answer 36,75 l.
Or, 36 l. 15 s. C's gain.

In the Resolution according to the new method. The Operations are (in brief) as follows, viz.

```
1648) 309,1 (,1875
      -------
      14420
      -----
      12360
      -----
       8240
      -----

     ,1875            ,1875
   A  432           B  1020
   ------           -------
     3750             37500
    5625             18750
   7500             ------------
   ------           B's is 191,2500 l.
A's gain is 81,000 l.

     ,1875
   C  196
   ------
    11250
   16875
   1875
   ------
C's is 36,7500 l.
```

The Proof.

	l.		*l. s. d.*
A's gain is	81,00	Or	81-00-00
B's ——	191,25		191-05-00
C's ——	36,75		36-15-00
Sum	309,00	Or,	309-00-00

The

The Fourth Example is in the Rule of Fellowship with Time.

SIX Merchants A, B, C, D, E and F Company, *viz.*

	l.	*s.*	*d.*		
A laid in	52	14	00	for	$6\frac{1}{2}$ months.
B	74	15	00		$7\frac{1}{4}$
C	83	03	00		$8\frac{1}{2}$
D	58	11	00		$12\frac{1}{2}$
E	112	15	00		$4\frac{3}{4}$
F	59	18	00		7

They gained (by the whole) 289 l. 14 s. 6 d. *Quere* each mans part of the gain according to his Stock and Time.

The several Stocks and Time being multiplied together, the respective products are as follows, *viz.*

A's Stock 52,7 l.
Time 6,5

2635
3162

product 342,55

B's Stock 74,75 l.
Time 7,25

37375
14950
52325

product 541,9375

C's Stock 83,15 l.
Time 8,5

41575
66520

product 706,775

D's Stock 58,55 l.
Time 12,5

29275
11710
5855

product 731,875

E's Stock

E's Stock 112,75 l. Time 4,75

56375
78925
45100

product 535,5625

F's Stock 59,9 *l.* Time 7

product 419,3

A's product	342,55
B's	541,9375
C's	706,775
D's	731,875
E's	535,5625
F's	419,3
Sum	3278,0000

In the Resolution according to the customary method, the several proportions are as follows, *viz.*

First, For A's gain.

If 3278 : 2089,725 *l.* : : 342,55. Answer 218,375625 *l.*
Or 218 l. 7 s. *6* d.0,6 f. A's gain.

Secondly, For B's gain.

If 3278 : 2089,725 l. : : 541,9375. Answ. 345,4831562 5*l.*
Or 345 l. 9 s. 8 d. 1,75 f. B's gain.

Thirdly, For C's gain.

If 3278 : 2089,725 *l.* : : 706,775. Answer 450,5690062 5 *l.*
Or 450 l. 11 s. 4 d. 2,5 f. C's gain.

Fourthly,

Fourthly, For D's gain.

If 3278 : 2089,725 l. 731,875. Answ. 466,5703125 l.
Or 466 l. 11 s. 4 d. 3,5 f. D's gain.

Fifthly, For E's gain.

If 3278 : 2089,725 l. 535,5625. Answ. 341,42109375 l.
Or 341 l. 8 s. 5 d. 0,25. E's gain.

Sixthly, For F's gain.

If 3278 : 2089,725 l. 419,3. Answ. 267,30375 l.
Or 267 l. 6 s. 0 d. 3,6 f. F's gain.

In the Resolution by the new method. The operations are, *viz.*

```
2378) 2089,725 (,6375        A's product  342,55
                                           ,6375
      12292                             171275
                                       239785
       24585                          102765
                                      205530
        16390
                             A's gain is 218,375625
           0

B's product  541,9375        C's product  706,775
               ,6375                       ,6375

          27096875                       3533875
         37935625                       4947425
        16258125                       2120325
       32516250                       4240650

B's gain is 345,48515625 l.  C's gain is 450,5690625 l.
```

D's pro-

D's product 731,875
,6375

3659375
5123125
2195625
4391250

D's gain is 466,5703125 l.

E's product 535,5625
,6375

26778125
37489375
16066875
32133750

E's gain is 341,42109375 l.

F's product 419,3
,6375

20965
29351
12579
25158

F's gain is 267,30375 l.

The Proof.

	l.	s.	d.	q.
A's gain is	218	07	06	0,6
B's	345	09	08	1,75
C's	450	11	04	2,3
D's	466	11	04	3,5
E's	341	08	05	0,25
F's	267	06	00	3,6
Sum	2089	14	06	0,00

The

The Fifth Example is in Alligation Alternate.

A Grocer hath four sorts of Sugar, *viz.* of 12 d. per l. of 10 d. per l. of 6 d. per l. and of 4 d. per l. of which he would make a Composition of 237 l. worth 9 d. per l. I demand how much he must take of each sort?

I shall here (for brevity sake) omit the divers ways of linking the several Rates or Prices of the Simples propounded, referring to other Authors that treat of Alligation at large, only shew the expeditious performance thereof by the general Rule preceding.

The prices here being disposed of and linked as in the Margent the sum of the several differences is 12.

	d.	differ.
9 d.	12	5
	10	3
	6	1
	4	3
	Sum	12

The Resolution hereof according to the customary method requires four proportions, (or the Rule of Three four times repeated) there being four quantities required, which proportions and operations are as follows, *viz.*

First, For the quantity of Sugar of 12 d. per l.

l.		l.		l.		l.
As 12	:	237	: :	5		Answer 98,75 or 98¾.

```
        237
          5
   12) 1185 (98,75
        105
         90
         60
```

Secondly,

Secondly, For the quantity of 10 d. per l.

	l.	*l.*		*l.*	
As	12 :	237	: :	3.	Answer 59,25 l. or 59¼.

```
      237
        3
   --------
12) 711 (59,25
   --------
     111
   --------
      30
   --------
      60
```

Thirdly, For the quantity at 6 d. per l.

	l.	*l.*		*l.*	
As	12 :	237	: :	1.	Answer 19,75 or 19¾ l.

```
      237
        1
   --------
12) 237 (19,75
   --------
     117
   --------
      90
   --------
      60
```

Fourthly, For the quantity at 4 d. per l.

	l.	*l.*		*l.*	
As	12 :	237	: :	3.	Answer 59,25 or 59¼ l.

In the Resolution hereof by the new method; the Operations are expeditiously performed as follows. 237 (the second number in the preceding proportions) being the dividend, and 12 (the first number) the Divisor.

I 12)

12) 237 (19,75
117
90
60

At 12 d. per l. 19,75
5
required 98,75 l.

At 10 d. per l. 19,75
3
required 59,25 l.

At 6 d. per l. 19,75
1
required 19,75 l.

At 4 d. per l. 19,75
3
required 59,25 l.

The Proof by Alligation Medial.

To make up this mixture there is required by the preceding work.

98,75 l. at 12 d. per l.	And at 12 d. per l.	98,75 l. is	1185,0 d.
59,25 — 10	10	59,25	592,5
19,75 — 6	6	19,75	118,5
59,25 — 4	4	59,25	237,0
237,00 the Sum.		the Sum is	2133,0

Also 237 l. at 9 d. per l. (the mean price) is 2133 pence.
Q. E. F.

The

The Sixth Example is in Alligation Alternate.

SUppose it were required to make a mixture of 155¼ gallons of Wine, *viz.* Of Canary at *6* s. and 5 s. 3 d. per gallon. Of Malligo at 4 s. and at 3 s. 9 d. per gallon. Of White-Wine, at 3 s. and at 2 s. 9 d. per gallon. In such sort that this mixture may be worth 4 s. *6* d. per gal. *Quære*, How much of each sort of Wine ought to be taken, to satisfie the condition of this question.

The several given prices of the respective sorts of Wine propounded being here disposed and linked as in the Margent, the sum of all their differ. (from the mean price given) is 108.

	d.		
d. 54	72	18 + 21	39
	63	6 +	15
	48	9	9
	45	9	9
	36	18	18
	33	18	18
		the Sum	108

The Resolution hereof by the Customary method (there being Six quantities sought) will require Six proportions, as follows, *viz.*

First, For the quantity of Canary at *6* s. or 72 d. per gal.

As 108 : 155,25 : : 39. Answer 56,0625 gal.

Secondly, For the quantity of *Canary* at *6*3 d. gallon.

As 108 : 155,25 : : 15. Answer 21,5625 gallons.

Thirdly, For the quantity of Malligo, at 4 s. or 48 d. per gal.

As 108 : 155,25 : : 9. Answer 12,9375 gallons.

Fourthly, For the quantity of Malligo at 45 d. per gallon.

As 108 : 155,25 : : 9. Answer 12,9375 gallons.

Fifthly, For the quantity of White-Wine at 3 s. or 36 d. gal.

As 108 : 155,25 : : 18. Answer 25,875 gallons.

Sixthly, For the quantity of White-Wine at 33 d. per gal.

As 108 : 155,25 : : 18. Answer 25,875 gallons.

In the Resolution according to the new method, 155,25 (being the second number and a common Factor in all the preceding proportions) is the dividend, and 108 the Divisor.

```
108) 155,25 (1,4375
     ------
      472
     ----
      405
     ----
      810
     ----
      540
     ----
        0
```

```
at 72 d.   1,4375
per gallon     39
          -------
          129375
          43125
          -------
required  56 0625 gal.
```

```
At 63 d.   1,4375
per gallon     15
          -------
           71875
           14375
          -------
requ.     21,5625
```

```
At 48 d.   1,4375
per gallon.     9
          -------
required 12,9375 gall.
```

At

At 36 d. per gallon	1,4375
	18
	115000
	14375
Required	25,875

Also at 45 d. per gal. required 12,9375 gallons the Factors being the same, as for the quantity of 48 d. per gall.

Likewise having the same Factors for the quantity of 33d. (as for that of 36 d. per gallon) there is required 25,875 gall.

Hence it appears (by the preceding work) there is required to make up the mixture propounded of each quantity as follows, as by proof is evident.

56,0625 gall.	of 72 d. per gall. which comes to	4036,5...d.
21,5625	of 63	1358,4375
12,9375	of 48	621,....
12,9375	of 45	582,1875
25,875	of 36	931,5...
25,875	of 33	853,875.
155,2500 gal.	at 54 per gal. (mean rate given)	8383,5000

The same method and operation hold in all the divers ways of linking or coupling the values or prices given whereby the quantities required are as often altered as the question admits of divers Alligations, and yet all true, and the mixture thereby produced will bear the rate propounded.

It may also be observed that what hath been here performed by Decimals in those Rules of plural proportion aforesaid. The same may likewise be effected by vulgar Arithmetick; but then you will be necessitated often to use vulgar Fractions, whose operations are for the most part much more tedious and troublesome then Decimals. Yet when it so hap-

pens (as sometimes it may) as that number which is a common Factor and usually the second term in the customary method, may be evenly divided by the first, there is no need of Decimals there being no fractions in the operation; as in this and such like Examples, *viz.*

Let it be required to make a mixture of 120 gallons of these four sorts of Wine. That is of Canary at 60 d. per gal. Of Malago at 54 d. Of Rhenish at 40 d. And of White-Wine at 34 d. per gallon, so as that the said Composition may bear the price of 48 d. per gallon.

40) 120 (3
0

d.	d.	diff.
48	60	14
	54	8
	40	6
	34	12
	Sum	40

120 (the common factor or the second term in the customary method) is here the dividend, and 40 (the first term) is the Divisor, and 3 is the quotient without a fraction, and the work is as follows.

Of Canary 14
3
required 42 gall.

Of Mallago 8
3
required 24 gallons

Of Rhenish 6
3
requires 18 gall.

Of White-Wine 12
3
required 36 gal.

		gall. d.	d.
Proof	Canary	42x60 =	2520
	Mallago	24x54 =	1296
	Rhenish	18x40 =	720
	W.Win.	36x34 =	1224
Sum		120x48 =	5760

But

But if that number which is the common factor and usually the second term cannot be evenly divided by the first. Then you must use either Decimals or vulgar fractions as in the following example being performed by vulgar fractions, *viz.*

I have 40 Bushels of Wheat at 60 d. per bushel to mix with a meaner sort of Wheat at 52 d. per bushel, and with one sort of Rye, at 39 d. per bushel, and with another sort of Rye at 33 d. per bushel. I demand how many bushels of these other sorts shall I commix with my 40 bushels of Wheat, to the end that the said mixture may be worth 42 d. per bushel.

This question is of that part of Alligation called (by Arithmeticians) Alternation partial, those preceding of Alternation Total.

d. 42	60	9
	52	3
	39	10
	33	18
		40

By this Alligation it appears that if 9 bushels of Wheat, at 60 d. per bushel, and 3 bushels of Wheat, at 52 d. per bushel, be mixed with 10 bushels of Rye, at 39 d. and 18 bushels of Rye, at 33 d. per bushel. This mixture may be afforded at 42 d. per bushel, and justly countervail the prices of the several sorts of grain to be mixed: but it is required to mix 40 bushels of Wheat, at 60 d. per bushel, with a certain quantity of each of the other sorts of grain premised, to find which by the customary method, the proportions and work is as follows, *viz.*

First, For the quantity of Wheat required at 52 d. per bush.

B. B. B.

As 9 W : 40 W : : 3 W : *facit.* $\frac{120}{9}$ or $13\frac{1}{3}$ bushels.

$$\begin{array}{l} 3 \\ \hline 9)\ 120\ (13\tfrac{3}{9} \\ \hline 30 \\ \hline 3 \end{array}$$

 Secondly,

idly, For the Quantity of Rye at 39 d. per bushel, say

	B.		B.	
W :	40	W : :	10	R : *facit* $\frac{400}{9}$ or $44\frac{4}{9}$ bushels.
	10			

9) 400 ($44\frac{4}{9}$

40

4

dly, For the quantity of Rye, at 33 d. per bushel. It holds

	B.		B.	
W :	40	W : :	18	R : *facit* $\frac{720}{9}$ or 80 bushels.
	18			

9) 720 (80

00

nce by the preceding operations, if with 40 bushels of at at 60 d. per bushel; you mingle $13\frac{1}{3}$ bushels of the grain at 52 d. per bushel, $44\frac{4}{9}$ bushels of Rye at 39 d. ushel, and 80 bushels at 33 d. per bushel, that mixture bear the mean price proposed, *viz.* 42 d. per bushel, h answers the condition of the question as by proof is ent.

40	bushels at 60 d. per bushel		is 2400 d.
or $13\frac{3}{9}$	52	$\frac{6240}{9}$	or $693\frac{1}{3}$
or $44\frac{4}{9}$	39	$\frac{15600}{9}$	or $1733\frac{1}{3}$
and 80	33		2640
m $177\frac{7}{9}$	42 d. per bush. (mean price)		$7466\frac{2}{3}$

In

In the Resolution of the last question according to the new method, 40 (the common factor in the preceding proportions) is the dividend and 9 the divisor, and the operation are as follows, *viz.*

9) 40 ($4\frac{4}{9}$ or $\frac{40}{9}$ is the common Factor. Therefore
$\frac{40}{9}$ multipl. by 3, gives $\frac{120}{9}$ or $13\frac{3}{9}$ the quant. of Wh. at 52 d.
$\frac{40}{9}$ ——— 10 ——— $\frac{400}{9}$ or $44\frac{4}{9}$ the quant. of Rye at 39
$\frac{40}{9}$ ——— 18 ——— $\frac{720}{9}$ or 80 the quant. of Rye at 33

Here the same quantities of each sort of grain are produced as before. And in Decimals the operations are as follows, *viz.*

9) 40 (4,444444, &c. is the quotient and comm. factor.

40

40, &c.

Note, that 40 is the quant. of Wheat at 60 d. per bush.
And 4,444444 x 3 = 13,333333 *&c.* bush. 52
also 4,444444x10 = 44,444444, *&c.* —— 39
and 4,444444x18 = 79,999999, *&c.* —— 33

quant. in the mixture is 177,777776 bushels as before found equal to $177\frac{7}{9}$ *prope.*

Thus much for Alligation Alternate which if well practised and understood is sufficient to enable the Reader for the Solution of any question pertaining to the said Rule, by this new method, as mixing of the mettals, Composition of medicines, and the like; for which see Sir *Jon.as Moore*'s Arithmetick, *Kersey* upon *VVingate*'s Arithmetick, and divers others.

The Reader may also Observe, That this compendious method for working the Rules of Fellowship and Alligation Alter-

Alternate, is not (to my knowledg) to be met with in any Authour, but in Mr. *Adam Martindale's* Appendix to his Surveying, he being the first Publisher thereof that I have hitherto seen. And I have the rather chosen to make this necessary digression, as well to accommodate Decimals to use in several Arithmetical operations, as also thereby the better to facilitate the work of the ensuing part of this Treatise to the understanding of the meanest capacity, so shall conclude Decimal Arithmetick and proceed to the Extraction of the square Root.

The

The Extraction of the Square ROOT.

1. TO Extract the Square-root of a given number, is to find such a number, which being multiplied by (or into) it self produces the given number; therefore,

2. A Square number is such a number, whose root may be exactly had; as being produced by the multiplication of any number into it self, and therefore is always commensurable to its root. Thus 25 is a Square number produced by the multiplication of 5 by 5 (which is also called Squaring of a number) where observe that 5 is called the root or side and 25 the Square.

3. Square numbers are either Single or Compound.

4. Single square numbers are such as are produced by the multiplication of any one Single figure into it self, and therefore every single square number is always less then, 100 (which is the square of 10.) So 25 is a single square number produced of 5 multiplied into it self, and 81 is a single square number produced by multiplying 9 into it self, *&c.*

5. All single square numbers with their respective roots or sides, are exprest in the following Table.

Roots or sides	1	2	3	4	5	6	7	8	9
Square numb.	1	4	9	16	25	36	49	64	81

6. When the root of any number less then 100 is required, which is not one of the single squares exprest in this Table; you are to take for his root, the root of that single square number in the Table, which being the next less yet comes nearest to the given number. Thus if the root of 18 is required, the greatest square in 18 is 16, and consequently

sequently 4 is the nearest Square root of 18, that can be had in whole numbers, (and how to find the fractional part to be annexed to the root shall be hereafter shown) the like is to be understood of any other number less then 100, not exprest in the Table.

7. Compound Square numbers, are such as are produced by the multiplication of any numbers consisting of more places then one, into (or by) themselves and are never less then 100 (which is the least compound square number.) Thus 625 is a compound Square number, produced by the Squaring of 25. and here 25 is the side or root, and 625 the square thereof. So any number greater then 9 being squared (or multiplied into it self) produces a compound square number.

8. The Root of any number under 100 may be easily discovered at sight, by the Table of single Squares according to the preceding directions, but to Extract the root of a compound Square number consisting of Integers, observe the following precepts.

First Point the given number, viz. place a point over the first figure towards the right hand, and omitting every other figure, place another over the 3d. 5th. and 7th. and so on according to the number of places, the figures thus distinguished are called points, and as many points as are in the number propounded for Extraction, of so many figures consists the root sought, if the number be rational or an exact compound Square.

Secondly, Draw a crooked line on the right hand of the number proposed (after the manner of division) for a quotient wherein to place the root.

Thirdly, Seek the greatest Square in the first point towards the left hand (by the Table of single Squares) place the root in the quotient and substract the said square from the first point, subscribing the remainder underneath the first point.

Fourthly, To this Remainder bring down the next point placing it on the right hand thereof, which call the Resolvend.

Fifth-

Fifthly, Double the quotient (or root of the first point) and place it on the left hand of the resolvend, distinguishing it therefrom by a crooked line thus,) which call the Divisor.

Sixthly, Seek how often this divisor (or double quotient) is contained in all the figures of the Resolvend except the last towards the right hand, (after the manner of division) place the answer in the quotient, and also on the right hand of the Divisor.

Seventhly, Multiply the Divisor with the figure last added by the figure lasted placed in the quotient, and substract that product from the Resolvend, subscribing the Remainder.

Eighthly, To this Remainder bring down the next point for a new Resolvend, and proceed therewith as with the first Resolvend, repeating the work of the 5th. 6th. and 7th. Rules (or precepts) foregoing and continuing so to do until the Extraction be finished.

The first Example.

Let it be required to Extract the Square-root of 625 the root whereof will be found to be 25 as by the following Operation.

First, The number being prepared by punctation (according to the first precept) stands thus, which consisting of two points, implyes the root sought to consist of two figures or places.

. .
625

Secondly, Drawing a crooked line on the right hand of the given number for a quotient, behind which to place the root, then the work will stand thus.

. .
625 (

Thirdly, The greatest Square in the first point, 6 is 4, whose root is 2, wherewith proceed (according to the 3d. precept) and the work stands thus.

. .
625 (2
4
———
Remain. 2

Fourth-

Fourthly, To the remainder 2 bringing down the next point 25, and proceeding therewith according to the 5th. 6th, and 7th. precepts, and the work stands as in the margent.

```
                 . .
                 625 (25
                 4
                 -----
divisor 45)  225 Resolv.
             225 Product.
             -----
              0
```

Here Note, That the points being all brought down and o remaining (after the last substraction) shews the operation is finished, and also that 625 is an exact compound square number, whose side or root is 25, as appears by squaring of 25 (being the proof the Extraction of the square-root) which produces 625 the number proposed.

```
  25
  25
 ----
 125
 50
 ----
 625
```

Note, also that when (the divisor with the figure last added, multiplied by the figure last placed in the quotient) the product exceeds the Resolvend the work is erroneous. To rectifie which place a less figure in the quotient, but if the remainder is greater then the next divisor place a greater figure in the quotient, wherewith proceed according to the 7th. precept, &c.

It may also be observed, that the reason of pointing every other figure, is because the square of any one single figure never exceeds two places.

The operations (at large) of 3 or 4 more examples will (I presume) make the Extraction of the Square Root of any rational (or Square) Number evident and intelligible enough, with little (if any) more Explication.

Note

More Examples.

Whats the square Root of 1369, The number prepared according to the 1st. and 2d. precept stands thus.

1369 (

The Operation of the 3d. precept being performed, the work stands thus,

1369 (3
9
Rem. 4

The operation of the 4th. precept being observed the work stands thus,

1369 (3
9
469 Resolv.

The operation of the 5th. precept being performed the work stands thus,

1369 (3
9
Divisor 6) 469 Resolv.

The operation of the 7th. precept being performed the work stands thus and the whole operation is as in the margent, which shews 1369 is a squ. number, composed of 37 squared.

1369 (37 Root.
9
div. 67(469 Resolv.
469 product.
0 Rem.

Whats

Whats the Square Root of 15129 (123 the root sought.
1

first divisor 22) 51 Resolvend
44 product

second divisor 243) 729 resolvend
729 product

0

Whats the Square root of 236196 (486 the root sought.
16

first divisor 88) 761 resolvend
704 product

second divisor 966) 5796 resolvend
5796 product

0

Whats the Square root of 13017664 (3608 the root requ
9

first divisor 66) 401 resolvend
396 product

second divisor 720) 576 resolvend
000 product.

third divisor 7208) 57664 resolvend
57664 product

0

Here

Here by the way observe, That when the divisor cannot be had in the Resolvend (according to the 6th. precept) then place a cypher in the quotient, and also on the right hand of the divisor, and then you may either remove the resolvend a step lower, and annex the next point thereto for a new resolvend, setting the new divisor also in the same line therewith, as in the operation of the last example. Or let both the resolvend and divisor remain, annexing the next point, as in the following operation of the last example, being again repeated.

```
                 . . . .
                13017664 (3608 the root required.
                9
                ----
divisor 66) 401 resolvend
            396 product
            ----------
divisor 7208) 57664 resolvend
              57664 product
              -----
                  0

                                  . . . .
Whats the square Root of 49126081 (7009 the root requ.
                         49
                         --
    first divisor 140) 12 resolvend
                       00 product
                       --
  second divisor 1400) 1260 resolvend
                       0000 product
                       ----
  third divisor 14009) 126081 resolvend
                       126081 product
                       ------
                            0
```

The last Example being again repeated, the operations performed in short, according to the preceding directions as follows.

```
           . . . .
          49126081 (7009
          49
divisor 14009) 126081 resolvend
               126081 product
                    0
```

But if there be a rational (or square) number proposed for Extraction, consisting either of Integers with Decimals annexed, or of Decimals alone, then point every other figure in the Decimals from unities place towards the right hand, as you do the integers towards the left hand, and as many points as are in the Decimals annexed or proposed, So many Decimal places are in the Root sought.

First, Examples of mixt Numbers commensurable to their Roots.

```
                              .  .  .
Whats the square Root of 3283,29 (57,3 the root sought.
                         25
            divisor 107) 783 resolvend
                         749 product
           divisor 1143) 3429 resolvend
                         3429 product
                            0
```

Whats

Whats the square root of 4495,7025 (67,05 the root requi.

```
          36
      ---------
 127) 895
      889
      ------------
13405) 67025
       67025
       --------
           0
```

Secondly, Examples of Decimals Numbers, Commensurable to their Roots.

Whats the Square root of ,1225 (,35 the root required.

```
      9
    -------
65) 325
    325
    -------
      0
```

Whats the square root of ,571536 (,756 the root required.

```
       49
     --------
145) 815
     725
     ---------
1506) 9036
      9036
      --------
         0
```

Also the square root of ,08254129 is ,2873.

And here observe, both in mixt numbers and Decimals alone that if the number of decimal places be not even, *viz.* two, four, six, eight, *&c.* places, then the number propounded is incommensurable to its square root.

Note also, when a decimal fraction is proposed for Extraction, which hath two or three cyphers, possessing the two or three first places next the left hand, cut off two of them with a dash of the pen, and place a cypher in the first place of the root, and if the given decimal have four cyphers before it, then cut them of, and place two cyphers in the root, and proceed to Extract the square Root of the remaining figures as is already taught.

Thus the Square root of ,004489 is ,067 see the Operation in the margent.

```
     . . .
   00|4489 (,067 the root.
      36
   -------
  127) 889
       889
      -----
        9
```

Also the square root of ,0000|351649 will (in like manner) be found to be ,00593.

Hitherto hath been spoken concerning the Extraction of the square root of any square or rational numbers, (whether whole, mixt, or decimal) *viz* such as are commensurable to their square roots.

But if when all the points in the given number are finished, there be yet a remainder after the last Substraction, such are called surd numbers (being incommensurable to their square roots) whose square roots, though they be inexplicable by numbers, yet may be obtained infinitely near by an approximation, according to the following directions, *viz.*

When a surd number is proposed for Extraction, consisting all of integers; proceed therewith as with a rational whole number till all the points are finished, and then there

there will always be a remainder, which not only shews that the number proposed is a surd number; but also that you have already found the greatest whole number that the Square root can consist of, and to find the decimal fraction to be annexed to the root found, to bring it nearer the truth, proceed thus, place two cyphers on the right hand of the remainder for a new resolvend, and find also a new divisor, wherewith proceed according to the 6th. and 7th. precepts, and there will be produced one figure more to be placed in the quotient, which is the first figure (or place of Tens) in the decimals, and so by a continual annexing of two cyphers to every last remainder, you may continue the Extraction to as many places of decimals as you please. For as many pairs of cyphers as are annexed so many places of decimals are in the root, which are to be distinguished from the integers by a comma.

Examples.

Whats the square root of 6968 (83,47 &c. the Root propo.
64

first divisor 163) 568 resolvend
489 product

second divisor 1664) 7900 resolvend
6656 product

third divisor 16687) 124400 resolvend
116809 product

Rem. 7591, &c.

Whats the square root of 56783 (238,29 &c. the root *prope*

```
               56783 (238,29 &c.
               4
             ------
        43)  167
             129
             --------
       468)  3883
              3744
              --------
      4762)   13900
                9524
                --------
     47649)    437600
               428841
               --------
                  8759, &c.
```

Here Note, That having found the integral part of the Square root of any Surd number, the remainder may be exprest in terms of a vulgar fraction thus, *viz.* Double the remainder for a numerator, and quadruple (or multiply by 4) the root found, whereunto add 1, for the denominator thereof, which vulgar fraction being annexed to the root found, that mixt number shall be the Square-root of the Surd number proposed, (*prope verum.*)

Thus, If the square-root of 18 were required, the integral part of the root is 4, and 2 remains, which doubled is 4 for the numerator, and 4 (the root found) quadrupled makes 16, to which add 1 and it is 17 for the denominator thereof. Hence I conclude that $4\frac{4}{17}$ is the neerest square-root of 18, that can be exprest in numbers, unless you use the method of approximation by decimals as aforesaid, by annexing a competent number of pairs of Cyphers, whereby the root of any Surd number may be obtained infinitely the truth.

But when a Surd number is proposed for Extraction, consisting of decimals alone, or of decimals annexed to integers, in such Cases it is most convenient to use the method of ap-

pro-

proximation aforesaid, regard had to the preceding Rules for that purpose, observing that if the decimal places be not even, they must be made so by annexing a cypher, for the decimals must always consist of two, four, six, eight, *&c.* places, before they be distinguished by points.

First, Examples of Mixt and Surd Numbers.

Whats the Square root of 1392,86 (37,31, *&c.* the root *firs.*
9

first divisor 67) 492 resolvend
469 product

second divisor 743) 2386 resolvend
2229 product.

third divisor 7562) 15700 resolvend
15124 product

Rem. 576, *&c.*

Whats the squ. root of 16780,493 — 16780,4930 (129,539

Here there must be a cypher annexed to the decimals (according to the Rule already delivered for that purpose) to make the number of decimal places even: and continuing the extraction after all the points in the given number are finished by annexing two cyphers, to make a nearer approach, the Square root will be, 129,539, *&c.*

1
22) 67
44
249) 2380
2241
2585) 13949
12925
25903) 102430
77709
259069) 2472100
2331621
Rem. 140479, *&c.*

Also the Square root of 155424,458 is 394,238 *fere.*

And the Square Root of 8368998,3458, is 2892,922 *fere.*

Secondly, examples of Decimals being surd quantities.

Whats the square root of ,75 (,866 the root required *prope.*

64

166) 1100
996

1726) 10400
10356

Rem. 44, *&c.*

Also the Square root of ,8125 is ,9013, *&c. fere.*

And the square root of ,39583 is ,62915, *&c. fere.*

If a vulgar fraction that is commensurable to its square root be proposed for Extraction; Extract the square-root of the numerator for a new numerator, and also of the denominator, for a new denominator, so shall the new fraction (thus found) be the square root of the fraction propounded; thus the square-root of $\frac{9}{16}$ is $\frac{3}{4}$; and the square root of $\frac{25}{36}$ is $\frac{5}{6}$.

But here observe, that if the fraction proposed for Extraction, be not in its least terms, it must first be reduced to its least terms, for it often happens that though the former be incommensurable to its square root, yet the latter may be commensurable. Thus if the square root of $\frac{12}{27}$ is required.

In its given terms the fraction in incommensurable to its square root, but being reduced to $\frac{4}{9}$ (its least terms) the square root is $\frac{2}{3}$.

Also the square root of $\frac{144}{441}$ is inexplicable in the given terms, but being reduced to $\frac{16}{49}$ (its least terms) the Square root is $\frac{4}{7}$.

I

If the vulgar Fraction given be incommensurable to its Square root, both in the given terms and also in any other terms that it is reduceable to. Then reduce the said vulgar Fraction into decimals consisting of even number of places. And then extract the Square root thereof by approximation according to the precepts already delivered.

Thus if the Square root $\frac{3}{4}$ be required its equivalent Decimal is ,75 whose Square root is ,86602, *&c.* therefore the Square root of $\frac{3}{4}$ is ,86602 *fere.*

Also if the Square root $\frac{13}{16}$ were required, its equivalent decimal is ,8125 whose Square root is ,9013 *fere*, *&c.*

Also if a mixt number is proposed for extraction whose fractional part is exprest in terms of a vulgar fraction (and not in decimals.) Reduce it into an improper fraction, and (if commensurable) extract the Square root of the numerator and denominator as before, observing the aforesaid caution of reducing the fractional part of the mixt number (or the improper fraction equivalent to the mixt number) into its least terms;

Thus if the Square root of $5\frac{4}{9}$ were required it will be $2\frac{1}{3}$ for the improper fraction equivalent to $5\frac{4}{9}$ is $\frac{49}{9}$ and the Square root of $\frac{49}{9}$ is $\frac{7}{3}$ or $2\frac{1}{3}$.

And (after the same manner) the Square root of $34\frac{33}{64}$ is $5\frac{7}{8}$. But if the improper fraction (equivalent to the mixt number proposed for extraction) be incommensurable to its Square root. Then reduce the fractional part of the mixt number into decimals, of an even number of places. And extract the Square root thereof by approach as before.

Thus if it were required to extract the Square of $126\frac{5}{8}$, the improper fraction thence resulting is $\frac{1013}{8}$ which because each term is incommensurable to its Square root, therefore reducing $\frac{5}{8}$ (the fractional part of the given mixt number) into decimals, (according to the last rule preceding) and the mixt decimal number thence resultiug will be 126,625 whose square root being extracted (according to the former rules) is 11,2528 *fere*. So that the Square root of $126\frac{5}{8}$ is 11,2528, &c.

Also the square root of $1392\frac{43}{50}$ being required, the improper fraction equivalent thereto is $\frac{69643}{50}$ which is incommensu-

menſurable to its Square root, therefore ,86 the decimal equivalent to the fractional part $\frac{43}{50}$ annexed to 1392 (the integral part of the given number) gives the mixt number 1392,86 whoſe Square root is 37,37 &c. which was required. The like underſtand of all other mixt Surd numbers.

Alſo you may obſerve (before we make an end of extraction of the Square root) that divers Authors expreſs the Square roots of all Surd quantities or numbers incommenſurable to their ſquare roots; whether whole, mixt or Fractional; by prefixing this character before the given number, *viz.* √ or √q. as implying that the Square root of the number or quantity before which it is prefixed is to be extracted. Thus the root of $5\frac{4}{9}$ may be thus expreſt, √. $5\frac{4}{9}$ or thus √q.$5\frac{4}{9}$. The root of ,75 thus √.75; or thus √.q.,75. The root of $\frac{13}{16}$ thus √.$\frac{13}{16}$, or thus √q.$\frac{13}{16}$, &c.

Note alſo, That the Square root of any fraction (leſs then 1 or unity) whether vulgar or decimal, is always leſs then the fraction propoſed for extraction. As ſuppoſe the ſquare root of $\frac{1}{4}$ or ,25 is required, the root will be $\frac{1}{2}$ or ,5, for $\frac{1}{2}$ ſquared is $\frac{1}{4}$ or ,5 ſquared is ,25. For it is evident that if a ſquare be made each ſide thereof a foot, then $\frac{1}{4}$ of that ſquare is 1 quarter of a ſquare foot, and the ſide of that ſquare (which contains $\frac{1}{4}$ of a ſquare foot) is $\frac{1}{2}$ a foot in length, and therefore the ſquare root of $\frac{1}{4}$ is $\frac{1}{2}$: The like is to be underſtood of any other fraction.

The Proof of the Extraction of the Square Root.

THE operation in the Extraction of the ſquare root may be proved divers ways, but the beſt and moſt uſual proof is by ſquaring the root found, and if the product with the remainder added (if there be any) be equal to the number propoſed for extraction, the operation is true, otherwiſe not: — or by the Croſs and caſting away the 9's thus. Caſt away all the 9's out of the number propoſed for extraction, and place the remainder over the Croſs. And alſo out

of

of the root found, placing the remainder on each side of the Cross, which remainder square, and thereto add the remainder after the extraction is ended (if there be any) out of which Sum cast away all the 9's. And this last remainder will be equal to that above the cross, if there be no errour in the operation; or by Division thus.

In any square number Divide the number proposed for Extraction, by the square root found: and the quotient will be equal to the said root without a remainder, if the operation is true.

In Surd numbers, the quotient will be the same as the root found, and also the remainder of the division equal to the remainder after the extraction is ended, if the operation is true.

As in the first Example in square numbers, where 625 is proposed to extract the square root thereof, which is there found to be 25, therefore seing 25 (the root found) being squared produces 625. I conclude the operation is true.

$$\begin{array}{r} 25 \\ 25 \\ \hline 125 \\ 50 \\ \hline \text{product } 625 \end{array}$$

The proof of the preceding Example by the Cross, &c.

The number proposed for Extraction is	625	4
The root found is	25	7 X 7
The remainder is	0	4

The

The proof by Division 25) 625 (25

125

0

The First Example in Surd numbers being examined according to the three foregoing Rules, for Proof, the operation thereof will be found true.

First by squaring the root found, and adding thereto the remainder after extraction.

The number proposed is——6968
The root found is————83,47
The remainder is ————7591

83,47
83,47

58429
33388
25041
66776

prod. 6967,2409
7591 Remaind.

Sum 6968,0000 the number proposed.

Secondly, By the Cross, &c.

The number given————6968

The Root found————83,47

The Remainder————7591

2
4 X 4
2

Thirdly,

Thirdly, by Division, &c.

83,47) 6968,0 (83,47

29040

39990

66020

Remainder 7591

Thus much for the Extraction of the Square root wherein I have been the more large, that it may be intelligible to ordinary capacities; and considering that I write to Learners and not the Learned, I hope my prolixity is pardonable, so shall not enlarge any further, referring the Reader to the third and fourth Cases of right Angled plain Triangles in the preceding Treatise of Arithmetical Trigonometry, for the use (of application) of the Square root in the Resolution of plain Triangles, being also subservient in the resolution of all the Cases, both in right and Oblique plain Triangles, according to the method delivered in the foresaid Treatise.

And to the end that this miscellaneous Tract may be of more general use then if here concluded, I shall in the subsequent part hereof, lay down the rise and Construction of several Tables, of general use in the Mathematicks in as intelligible terms, and with as plain directions, as I can possibly, considering my limits, *viz.*

First, *Of the Natural Sines, Tangents and Secants, with their rise, and the Construction or making Tables thereof.*

Secondly, *Of the Logarithms their rise, and Construction.*

Thirdly, *Of the Logarithmical (or Artificial) Sines, Tangents, and Secants.*

The Definition of a CIRCLE and its Parts.

BY way of Introduction, as also for method sake, it may be proper to begin with the Definition of a Circle and its parts, before we proceed to the Definition and Construction of the Natural Sines, Tangents and Secants.

Figure 22, Definition 1. A Circle is a plain figure, contained under one term or line called the Circumference, and by some the Perimeter or Periphery; as a b c d.

2. The Center of a Circle is that point in the very midst thereof (as at e) from which all right lines (called Radius's) drawn to the Circumference, are equal to one another, as e a, e b, e o, e c, e d.

3. The Diameter of a Circle, is a right line drawn from one side of the Circle, to pass through the center to the other side thereof, dividing the circle into two equal parts, as a c and b d, &c.

4. The Semi-Diameter or Radius of a Circle, is just one half of the Diameter, being contained between the center and the circumference, as ea, eb, eo, ec, ed, &c.

5. A Semi-Circle is half the circumference of a whole circle, being limited by the diameter as a b c or a d c.

6. A Quadrant is the fourth part of the circumference of a whole circle, or half of a semi-circle, and is limited by the two diameters a c and bd crossing each other at right Angles in the center, as the Arches ab, bc, cd, and da, each being a Quadrant.

7. An

7. An Arch of a circle, is some part of the circumference limited by two points called the terms or extremities thereof, as the Arches ao, bo, or oc.

8. The circumference of every circle is supposed to be divided into 360 equal parts called degrees, each degree, subdivided into 60 equal parts, called minutes; and again each minute into 60 equal parts called seconds, and so on; this being called the Sexagesimal division. Hence it follows,

9. A Semi-circle contains 180 degrees, and a Quadrant 90 degrees.

10. The measure of an Angle is an Arch of a circle intercepted between the two sides containing the Angle being described on the Angular point as a center, and as many degrees as the Arch contains, so much is the Angle; so the measure of a right Angle (as aeb, bec, &c.) is always 90 deg. or a Quadrant.

11. An Acute Angle (as beo) is always less; and an Obtuse Angle (as aeo) greater then a right angle.

12. The Complement of an Arch or Angle to a quadrant, is so much as it wants of 90 deg. as the Arches bo and oc, being both 90 deg, are thereof the Complement of each other to 90 deg. But the Complement of an Arch or Angle to a Semi-circle is so much as it wants of 180 deg. as the Arches ao and oc are the Complement of each other to 180 deg. and both together equal to 180 deg.

Now forasmuch as the Ratio, or proportion of Arches one to another, as also of an Arch to a right-line is yet unknown; and because the Angles of plain Triangles, as also both the Angles and sides of Spherical, are measured by Arches of a circle; therefore the proportion of all the parts of a Triangle one to another cannot be determined, in order to their calculation; unless those Arches be first reduced to right-lines, and the length of each defined according to an assigned Radius.

The

The Right-Lines of a Circle applyed to the measuring of Arches and Angles, are Chords, Sines, Tangents, Semi-Tangents and Secants.

Such as are now generally used in the Calculation of Triangles; are Sines, Tangents, Secants, and sometimes Versed Sines.

Figure 22. Def. 1. The Chord or Subtense of an Arch or Angle is a right-line drawn within a circle dividing the circumference into parts, and is a Chord to them both. So o k is a chord of the Arch o c k, and also of the Arch o a k, the Complement of o c k to 360 deg. Likewise o c is a Chord of the Arch o w c (or the Angle o e c) as also of the Arch o a c, which is the Complement of o w c to 360 deg. The greatest chord is a c or b d the diameter of the circle, and is the chord of half the circumference.

2. A Sine is either Right or Versed.

The right Sine of an Arch or Angle, is a right-line drawn within a Semi-circle from one end or term of an Arch perpendicular to the Diameter; and is a Sine to two Arches both equal to a Semi-circle, and therefore are the Complements of each other to 180 deg. as o n is a Sine of the Arch o c (or the Angle o e c) and also of the Arch o b c (or the Angle o e a) the Complement of o c to 180 deg. Or, the Sine of an arch is half the chord of twice that arch. Thus o n the Sine of the arch o w c is half of o k the chord of the double arch o c k. The greatest Sine, is e b the Radius or Semi-Diameter of the circle, and is called the whole Sine, being the Sine of a Quadrant or 90 deg.

3. A Versed Sine, is that part of the diameter contained between the right Sine of an Arch and the circumference; thus n c is the Versed Sine of the Arch o c (or k c) and a n the Versed Sine of the Arch o b a (or k d a.)

4. The

4. The Tangent of an Arch or Angle is a right-line perpendicular to the end of the Diameter, & touching the circle without, being limited by a right-line preceding from the center, by the end or term of the Arch: thus c m is a Tangent of the Arch c o (or the Angle c e o) as likewise of the Arch o b a (or the Angle o e a) the Complement of o c to 180 degrees.

5. The Secant of an Arch or Angle, is a right-line drawn from the center of the circle, cutting the circumference in one of the extremities of the Arch and continued till it meet with the Tangent of the same Arch, as e m is a Secant of the Arch o c (or the Angle o e c) and also of the Arch o b a (or the Angle o e a) the complement thereof to 180 d.

6. A Semi-Tangent is the Tangent of half an Arch or Angle, and not half the whole Tangent of an Arch, thus the Semi-Tangent of the Arch o c, is c F (the whole Tangent of c w which is half the Arch o c) and not half of c m (the whole Tangent of the Arch o c.)

7. The Co-sine of an Arch or Angle less then a quadrant, is the Sine of that Arches complement to 90 d. the like understand of the Co-Tangent or Co-Secant. Thus T o is the Co-Sine, bR the Co-Tangent, and eR the Co-Secant of the Arch cwo being also the Sine, Tangent and Secant of the Arch bo the complement of oc to 90 d.

8. The Co-sine, Co-Tangents, &c. of a Arch greater then a Quadrant, is the Sine, Tangent, *&c.* of the excess of that Arch above a quadrant or 90 d. Thus the cosine, co-tangent and co-secant of the Arch oba is To bR and eR.

9. By the preceding definitions, it eviden t that a Chord is common to two Arches, which together make a whole Circle, and are therefore the complements of each other to 360 d. and likewise a sine, tangent and secant is common to two Arches, which together are equal to a Semi-circle, and are the therefore the complements of each other to 180 d.

Thus much for the definition or description of Right-lines applyed to Arches, next follows the Construction or making of those natural Lines.

The Construction or making of the Natural Sines, Tangents and Secants is either Linear or Tabular.

THE Linear Construction is the making of those Lines, as they are usually plac'd upon the Marriners plain Scale, and may be performed Geometrically, as follows.

Figure 23. First, Describe a circle as abcd of a convenient Radius, which divide into four quarters or quadrants by two Diameters, ac and bd crossing each other at Right Angles in the Center.

2. For the Line of Chords; draw the chord Line of the quadrant bc; and divide the arch thereof into 9 equal parts, which mark with 10, 20, 30, &c to 90 from c to b, then with the one foot of your compasses resting in c as a center, with the other transfer the several divisions from the arch to the Right Line b c, and number it also with 10, 20, 30, &c. corresponding those in the arch, so shall bc be a line of Chords to every 10 deg. and if every 10 in the arch be again divided into 10 equal parts, and in like manner transferred to the chord Line, will give all the intermediate or single degrees; but it may suffice if every 10 in the Right-line be divided into 10 equal parts.

3. For the Line of Right Sines; From the points of the several divisions 10, 20, 30, &c. in the arch cb, draw parallels to cb cutting ec, so shall ec be divided into a line of right Sines which number with 10, 20, 30, &c. to 90 from e to c, the single degrees may be had, by dividing every

10

10 in the arch into 10 equal parts, and drawing parallels as before, but if every 10 in the in the right line ec, be divided into 10 equal parts it will suffice without any sensible error.

4. If those several divisions in the right line ec be numbred with 10, 20, 30, &c. from c to e it will be a line of Versed Sines to 90 deg. and may after the same manner be continued from e to a (the other half of the Diameter) to 180 degrees.

5. For the Line of Tangents; On one end of the Diameter as at c, raise the perpendicular cm to touch the circle without, which continue to a sufficient length; and by the edge of a Ruler laid from the center e to the several divisions 10, 20, 30, &c. in the arch of the quadrant cb. Divide the line cm into those several parts 10, 20, 30 &c. corresponding the divisions on the arch; hereby is cm unequally divided, and becomes a line of whole Tangents.

6. For the Line of Secants; Set one foot of the Compasses in the Center at e, and with the other transfer those several extents to the graduations in the line cm, to the right line eR, which mark also with 10, 20, 30, &c. correspondent to the graduations in the line cm, so shall eR be a line of Secants. The single degrees on the lines of Tangents and Secants ought to be graduated by the same reason from the single degrees of the arch as every 10 is especially those above 40 d. for then they increase fast and also very unequally.

7. For the Line of Semi-Tangents. By the edge of a Ruler laid from a to the several Divisions 10, 20, 30, &c. in the quadrant c b. Divide the line eb into the several parts 10, 20, 30, &c. to 90 d. corresponding with those in the arch, so shall eb become a line of Semi-Tangents, the single degrees may be divided like as the 10's or if every 10 in the line eb be equally divided into 10 it may serve.

8. There is also pertaining to the plain scale several other lines, as lines of equal parts of several lengths fitted to several Radius's of Chords, also a line of Rumbs or points of the Compass, a line of miles of Longitude and some has a line of hours each whereof (though it be some digression from my present subject) I shall give a brief hint of their Construction.

9. For the Lines of Equal parts; There needs no directions.

10. For the Line of Rumbs. Divide the arch of the quadrant ab into 8 equal parts, then one point of the Compasses resting in a, with the other transfer the several extents to those divisions unto the right line ab which number with 1, 2, 3, &c. to 8 from a to b, so is ab a line of Rumbs or points of the Compass.

11. For the Line of Hours. Divide the arch of the quadrant ad into 6 equal parts, and from a as a center transfer them, (in like manner) to the right line ad which will be a line of Hours, to be used with the foresaid line of chords.

12. For the Line of Longitude. Draw the right lines d6 parallel to ec and c6 parallel to ed; so is the square decb constructed, divide the line d6 into six equal parts, and draw the lines 5 e, 4 d, &c. parallel to de whereby the arch of the quadrant cd is divided into six unequal parts; Then placing one point of the Compasses in c with the other transfer the several extents to the points edc, &c. in the arch to the right line cd, so shall cd be a line of Longitude, to every 10 miles or minutes, and all the intermediate or single parts, ought to be divided like as the 10's is by dividing the several parts of the line d6 into 10 equal parts, and drawing parallels for each, &c. This line of Longitude is to be used with the line of chords aforesaid and serves to find how many miles or minutes, in any parallel of Latitude alters (or answers to) a degree of Longitude in the equinoctial. But as to the several uses of those lines aforesaid my limits will not permit to treat of here, being so frequent and general in the practical parts of the Mathematicks. I shall therefore refer to such as have already well applyed them to use, as *Collins Plain Scale*, *Sellers Practical Navigation*, and several Others. Only observe,

That those lines being all made to one Radius (as those preceding are) they may be used jointly one with another and not otherwise; and if they be made all to correspond with the same Radius, then the chord of 60 deg. the Tangent, of 45 d. the Semi-Tangent, and Sine of 90 d. are all equal one to another, and also equal to the Semi-diameter or Radius.

Note,

Note alſo, That if the Radius be divided into any number of equal parts, but moſt conveniently 10, 100, &c. and if a line of equal parts be made correſponding therewith of a ſufficient length, you may thereby meaſure the length of the Chord, Sine, Tangent, &c. of any arch, and find how many of thoſe parts of the Radius are contained therein, but this being not accurate enough for Arithmetical calculations, it is therefore abſolutely neceſſary that thoſe right lines (eſpecially ſuch as are commonly uſed in the calculation of Triangles) be determined to greater accuracy in numbers, to an aſſigned Radius and Tables thereof being accordingly made, are called the Cannon of Natural Sines, Tangents and Secants. The Conſtruction whereof follows.

To make the Tables of Natural Sines, Tangents and Secants.

FIrſt aſſign the Radius to be an unit with a competent number of cyphers annexed thereto, and thereby the Radius will be decimally divided, *viz.* into 1000, 10000, 100000, &c. equal parts, which kind of diviſion is the moſt commodious in all Arithmetical operations. Now to determine or expreſs how many of thoſe parts are contained in the Sine, Tangent, or Secant of an arch is the conſtruction making of thoſe Tables, which to perform divers Learned Mathematicians have largely writ thereof, as *Snellius*, *Pitiſcus*, *Briggs*, Dr. *Newton* and others, but the moſt facile and expeditious way is that publiſhed by Mr. *Collins* in his book of the Plain Scale (before mentioned) pag. 112. and the excellent ſinical proportion alluding thereto was (as he there ſignifies) attained unto and communicated to him by

Mr. *Dary*, both able Mathematicians of our own Countrey; the same being also afterwards published by M. *Dary* himself in his Miscellanies, pag. 5. which is the following effect.

In a rank of Arches equally differing.

As the Sine of any Arch in that rank: Is to the sum of the Sines of any two arches equally remote from it on each side : : So is the Sine of any other arch in the said rank : To the sum of the Sines of the two arches next to it on each side, having the like common difference. Also the same holds if the progression be interrupted, *viz*. when there are two ranks, whereof the first have the same common difference as the last, but not retaining the like difference between the last arch of the first rank and the first of the latter rank. Thus if the two ranks were 10, 14, 18 d. &c. and 35, 39, 43 d. &c. here each rank hath the same common difference but not in a progressive order, the like understand of any other two ranks of the same nature, this is premised in order.

To make the Tables of Natural Sines.

1. LEt the proportion of the Diameter to the circumference of the circle be taken for granted to be as 113 to 355, or in larger numbers. As 1 to 3,14159, &c. (for which see Dr. *Wallis*'s Algebra pag. 46.)

2. The Chord, Sine, or Tangent of 1 min. doth insensibly differ from the length of the arch to which it belongs, and therefore the length of the arch of 1 min. may (without any errour) be taken for the Sine thereof.

3. The

3. The Radius 100000, &c. being doubled gives 200000 &c. the diameter, which done, find the circumference in the same parts by the 1. Thus

As 1 : 3,14159, &c. : . 200000,00 &c. To 628318,53, &c, which is the length of the circumference in like equal parts of the Radius.

4. It is demonstrative, That such proportion as the circumference of one circle has to another such have thier Diameters, Sines, Degrees, &c. of like Arches one to another, and the contrary; therefore

5. The whole circumference being found as before to contain 628318, &c. equal parts of the Radius 100000, and if the Rad. be taken to be 10000000 then it is 628318,53 &c. which done, find by the 4 aforesaid how many of the like parts is contained in the arch of 1 min. Thus, As 21600 : (the min. in 360 deg. the whole circumference of a circle) Is to 628318,53 (the circumf. found in equal parts of the Rad.) : : So is 1 min. to the length of it corresponding arch : therefore dividing 628318,53 by 21600 the quotient is 29,0888, &c. or 29, for the Sine of 1 min. to the Radius 100000 : or the Radius being 100000,00, the Sine is 29,09 *fere.*

21600) 628318,53 (29,0888, &c.

1963

1918

1905

177, &c.

6 The Sine of 1 min. being thus obtained, the Cosine thereof (or Sine of 89 deg. 59 min.) may be thus had, *viz.* From the square of Radius, substract the square of the Sine of 1 min. The square root of the remainder is the Cosine thereof, thus the squ. of the Rad. (100000,) is 1 0000000000, and the square of 29,09 (the Sine of 1 min.) is 846,2281. Therefore

From the ſquare of Radius	10000000000,0000
Subſt. the ſquare of the Sine of 1 m.	846,2281
Rem. the ſqu. of the Co-S. of 1 m.	9999999153,7719

Extr. the ſqu. Root of 9999999153,7719 (99999,995, &c.
81

189) 1899
1701

1989) 19899
17901

19989) 199891
179901

199989) 1999053
1799901

1999989) 19915277
17999901

19999989) 191537619
179999901

11537718, &c.

The Co-ſine of 1 min. (or the Sine of 89 degrees 59 min.) is 99999,996 to the Radius 100000,000.

By what is premiſed in the 1, 2, 3, 4, 5, and 6. foregoing and the preceding operations, the manner of attaining the Sine and Co-ſine of 1 min. is plain.

And from the Sinical proportion before-mentioned of a rank of arches, &c. with ſome further explication thereof, all the reſt of the ſines in the quadrant may be calculated, as follows.

Having

Having obtained the Sine and co-sine of 1 min. we may by what is already known propose two ranks of arches, each rank to consist of three arches, and to have the like common difference though not in a progressive order but interrupted as aforesaid, which may be applyed to the preceding proportion. Thus,

In a Semi-circle, take the Radius for the middlemost of three arches in one rank; then the two arches on each side thereof must be, one to exceed, and the other be less then the quadrant, each by 1 min. the same Sine being (already known and) common to both arches, and therefore (applicable to the aforesaid proportion) must be doubled, this rank is at the end or last part of the quadrant. ——And for the other rank make the sine of 1 min. the middlemost of the three, and the two arches on each side thereof is 2 min. and 0 min. this rank is in the beginning of the quadrant. And these two ranks have the same common difference, *viz.* 1 min. Now the sines of all those arches are known except that for 2 min. which to find by the former proportion it follows, That As the Radius is to the double of the co-sine of 1 m. : : So is the sine of 1 min. : To the sine of 2 min. and 0 min. And from the same reason (having thus got the sine of 2 min.) retaining the two first terms it holds. So is the sine of 2 min. To the sum of the sines of 3 min. and 1 min. from which substracting the sine of 1 min. there remains the sine of 3 min. and then it will be, so is the sine of 3 min. to the sum of the sines of the 4 min. and 2 min. Again so is the sine of 4 min. : To the sum of the sines of 5 min. and 3 min. &c. and so proceed (observing the like order) from the beginning of the quadrant upwards.

It also follows (from the former proportion). for finding the sines near the end of the quadrant first and so to run downwards, that,

As Radius is to the double of the Co-sine of 1 min. : : So is the co-sine of the said min. To the sum of the co-sines of 2 min. and 0 min. The co-sine of 0 min. is the Radius, therefore from this fourth term found substract the Radius

the

there remains the co-sine of 2 m. or sine of 89 deg. 58 min. Then continuing—— So is the Co-sine of 2 min. To the sum of the co-sines of 3 min. and 1 min. from which substract the co-sine of 1 min. (the sine of 89 deg. 59 min.) rem. the co-sine of 3 min. (the sine of 89 deg. 57 min.)

Again, So is the co-sine of 3 min. To the sum of the co-sines of 4 min. and 2 min. and so on by the like orderly process.

This excellent proportion produces an easie operation for the first term being Radius division is avoided, and because the second term varies not (being a common Factor in every proportion) therefore having the Multiple's thereof, by all the 9 digits, (in a Table) ready at hand. The whole calculation may then be performed by Addition and Substraction.

A Table of the Multiple's of (199999,99155, &c.) the double of the co-sine of 1 min. (being the second term and common Factor) for the more ease and speed in calculation and may be continued further at pleasure.

1	199999,99155, &c.
2	399999,98311
3	599999,97466
4	799999,96622
5	999999,95777
6	1199999,94933
7	1399999,94088
8	1599999,93244
9	1799999,92399

The Operations, beginning with the Sine of 2 min. and so on to 3 min. 4 min. 5 min. &c. (in the first part of the quadrant) proceeding upwards by the like process, as follows.

For

For the Sine of 2 Minutes.

As 100000,00 : 199999,9915 : : 29,0888 : 58,18 *fere.*

```
                 29,0888
          ---------------
            159999993244
           159999993244
          159999993244
        1799999923996
        399999998311
        ------------------
1000,&c.) 58,17|759,754339084 (58,18 fere the sine of
          -----------------          (2 min.
```

For the Sine of 3 minutes.

As 100000,00 : 199999,9915 : : 58,1776 : 116,3552 *fe.*

```
                 58,1776
          ---------------
            119999999493
           139999999409
          139999999409
          199999999915
        159999999324
        9999999578
        ------------------
From      116,35|519,508894835 (the sum of the Sines of
Substract  29,0888 the Sine of 1 min.      (3 m. & 1 m.
          ----------
Remains    87,2664 the Sine of 3 minutes.
          ----------
```

For

For the Sine of 4 minutes.

As 100000,00 : 199999,9915 : : 87,2664 : 174,5328
87,2664 (fere.

7999999662
11999999493
11999999493
3999999831
13999999409
15999999324

From 174,53|279,26264892 the sum of the sines of 4 m.
Substract 58,1776 the sine of 2 min. (and 2 min.

Remain 116,3552 the sine of 4 minutes.

(and 3 m.
In like manner is found 232,7104 the sum of the sines of 5 m.
From which Substract 87,2664 the sine of 3 m.

Remains 145,4440 the sine of 5 min.

Also 290,888 is the sum of the sines of 6 min. and 4 m.
Subst. 116,355 the sine of 4 minutes.

Remain 174,533 the sine of 6 minutes, &c.

These Examples may suffice the same process being to be observed throughout the whole calculation.

And here Note, That although the Sines here found (be in effect) equally differing in such sort that the Sine of 2 m. is double, the sine of 3 m. triple the sine of 4 m. quadruple, &c. to the sine of 1 min. yet in prosecuting the work this kind of progression will quickly cease.

To

To find the Natural Sines beginning at the end of the Quadrant with the Arch 89 *d.* 58 *min. the Sine whereof is the Co-sine of* 2 *min. and descending successively, &c.* The Operations are as follows.

As 100000,00 : 199999,99,15 : : 99999,9957
99999,99,57

13999999405
9999999575
17999999235
17999999235
17999999235
17999999235
17999999235
17999999235
17999999235

From 199999,98|290,00003655
Substr. 100000,00 the co-sine of 0 min.

Remain 99999,98,3 the co-sine of 2 m.(Sine of 89 d.58m.

Here 199999,983 is the sum of the co-sines of 2 min. and 0 min. or the sum of the sines of 89 deg. 58 min. and 90 d. Therefore Radius substracted therefrom leaves 99999,983 the sine of 89 degrees 58 minutes, or the co-sine of 2 minutes.

For

For the Sine of 89 *deg.* 57 *min.the Co-sine of* 3 *min.*

As 100000,00 : 199999,992 : : 99999,983 : 199999,958 (&c.

99999,983

599999976
1599999936
1799999928
1799999928
1799999928
1799999928
1799999928
1799999928

From 199999,95|800,000136 the sum of the cosines of (3 m. & 1 m.
Substr. 99999,996 the cosine of 1 min.

Remain 99999,962 the co-sine of 3 min. (Sine of 89 deg. (57 min.)

By the like method the sine of 89 d, 56 m. is 99999,933
And the sine of 89 deg. 55 min. is 99999,894
And the Sine of 89 deg. 54 min. is 99999,845

These Examples with (regard to the preceding directions) are sufficient to illustrate the manner and process, either for calculating a whole Table of Natural Sines *de novo*, or to examine one already made.

But if this progress by 1 min. at a time be thought too tedious or slow, the more to expedite the work it may suffice to find the sines by this method, to every 10 minutes, and then to supply the intermediate sines by addition or substraction of their proportional parts of the differences of those so found. And in order hereunto it will be convenient for every sine found at the beginning of the quadrant, (in proceeding upwards) to find the sine of its complement.

To

To the end that we may have three equi-different arches or (which is all one) the sine of an arch doubled near the end of the quadrant, differing from the Radius 10 min. and so still retain Radius for the middlemost arch and first term, and also the co-sine of 10 m. doubled for the second term and common Factor, throughout the rest of the work: And then having already the sine and co-sine of 5 min. it will run thus (agreeable to the former proportion) to find the sines and co-sines of 10 m. 20 m. 30 m. &c. As Radius; to the double of the co-sine of 5 min. so is the sine of 5 min. To the sum of the sines of 10 m. and 0 m. And so is the co-sine of 5 min. to the sum of the co-sines of 10 m. and 0 m. And then As Radius, to the double of the co-sine of 10 min. So is the sine of 10 m. to the sum of the sines of 20 m. and 0 m. And so is the co-sine of 10 m. To the sum of the co-sines of 20 m. and 0 m. And now the co-sine of 10 min. doubled may be made the second term (and common Factor) for finding all the rest of the sines to every 10 m. throughout the quadrant. thus, As Radius; To double the co-sine of 10 m. so is the sine of 20 m.: To the sum of the sines of 30 m. and 10 m. And so is the co-sine of 20 m. to the sum of the co-sines of 30 m. and 10 m. And continuing, so is the sine of 30 m. to the sum of the Sines of 20 m. and 40 min. And so is the co-Sine of 30 m. to the sum of the co-Sines of 20 m. and 40 m. and so proceed till the work is finished. Two or Three examples will make it plain.

For the Sine of 10 *m.*

As 100000,00 : 199999,788 : : 145,444 : 290,89 *fere*

```
            145,444
        ----------------
            799999,152
           799999152
          799999152
         999998940
        799999152
       199999788
  ------------------------                (10 min.
10000,00) 290,88|769,165872 (290,8877 fere, the Sine of
```

For the Co-sine of 10 *min. Sine of* 89 *deg.* 50 *min.*

As 100000,00 : 199999,788 : : 99999,894 :
Facit 199999 5772, &c. the sum of the co-sines (of 10 m. & 0 min.
From which sub. 100000,0000 the Rad. or co-sine of 0 min.

Remains 99999,5772 the sine 89 d. 50 m. or co-sine (of 10 min.
Which doubl. is 199999,1544 for a common Factor.

A Table of the Multiples, of 199999,1544 which (being made a common, factor to find all the rest of the sines by to every 10 min.) will be of the like use as the former, in facilitating the work.

1	199999,1544
2	399998,3088
3	599997,4632
4	799996,6176
5	999995,7720
6	1199994,9264
7	1399994,0808
8	1599993,2352
9	1799992,3896

For the Sine of 20 *Minutes.*

As 100000,00 : 199999,1544 : : 290,8877 :
facit 581,77294, &c. the sine of 20 minutes, or (581,773 *fere.*

For

For the Co-sine of 20 m. (the sine of 89 d. 40 m.)

As 100000,00 : 199999,1544 : : 99999,5772 :

Facit 199998,3088,&c. the sum of Cosines of 20 and 0 m.

From which subst. 100000, &c. the Cosine of 0 min.

Rem. 99998,3088, or 99998,31, *fere*, the sine of (89 d. 40 m.

For the Sine *of* 30 *min.*

As 100000,00 : 199999,154 : : 581,773 :

Facit 1163,5411,&c. sum of the Sines of 10 m. 30 m.
From which sub. 290,8877 the Sine of 10 min.

Rem. 872,6534 the Sine of 30 min.

For the Co-sine of 30 min. Sine of 89 deg. 30 min.

As 100000,00 : 199999,154 : : 99998,308 :

Facit 1999995,7719 the sum of the Co-sines of (10 m. and 30 min.
From which subst. 99998,5772 the Co-sine of 10 min.

Remain. 99996,1947 the Sine of 89 deg. 30 m.

Also the Sine 40 m. is 116,5265;
And the Co-sine 99993,235 *fere.*

After the same manner proceed to find the Sines of 40 m. 50 m. 1 deg. 1 deg. 10 m. and so on successively throughout the quadrant.

Or, Having made the Sines and Co-sines to every 10 m. as far as 5 deg. (which are also the co-sines and sines respectively to every 10 m. from 85 d. to 90 d.) find the sine and co-sine of 10 d. and then proceed by 10 deg. at a time and find, the Sines of 20 d. 30 deg. and so on to 70 d. by the former proportions, which done then by the help of those sines already known, (*viz.* the first and last 5 deg. to every 10 m. and also the sines of all the inclusive arches to every 10 deg.) The sine of any other arch whatsoever may be readily calculated, provide it fall on an even number of 10 min. without having respect to the successive order aforesaid, for then no arch in the quadrant can be proposed but it will be within 5 deg. or less of some known sine, and by consequence the sine and co-sine of the arch of difference between the arch of a known sine and any arch proposed, will be also known. Therefore the arch of a known sine being made the middlemost of 3 equi-different arches, the sines of any (one or) two arches 5 deg. or less equally remote therefrom on each side may from thence be found by the two following proportions, *viz.*

1. As the Radius : Is to the co-sine of the arch of difference : : So is the sine of the mean or middle arch : To the half sum of the Sines of the extream arches. Again

2. As the Radius : Is to the Sine of the arch of difference : So is the co-sine of the middle arch : To the half difference of the sines of the said extream arches.

The half difference of those sines added to their half sum, gives the sine of the greater; and substracted leaves the sine of the lesser of those extream Arches. Or, the whole difference added to the sine of the lesser makes the greater; or substracted from the sine of the greater, makes the lesser of the sines of the extream arches.

Or, The substance of these two proportions may (for a general Rule) be thus exprest as a consectary therefrom.

Multiply the sine of the mean arch, and the co-sine of the arch of difference together, also multiply the co-sine of the middle arch, and the sine of the arch of difference together.

The

The sum of those two products divided by the Radius, gives the sine of the greater, and the difference of those two products divided by the Radius gives the sine of the lesser of the extream arches.

First Example:

The sine of 45 deg. being found to be 70710,678, To find the sines of 44 deg. 40 min. and 45 deg. 20 min. here 20 min. is the common difference whose sine is 581,773 and co-sine 99998,308.

1. By the former of the two last proportions, To find the half sum of the sines of the extream arches 44 deg. 40 min. and 45 deg. 20 min.

As 100000,000 : 99998,308 : : 70710,678 :

Answ. 70709,481 the half sum of the sines of the extream arches.

2. By the latter of the two proportions aforesaid, To find the half difference of the sines of the extream arches.

As 100000,000 : 581,773 : : 70710,678 :

Answ. 411,375 the half difference of the sines of the extream arches.

The ½ sum is —— 70709,481 } of the sines of 45 deg. 20 min,
The ½ differ. is — 411,375 } and 44 d. 40 m.

Sum is	71120,856	the sine of 45 deg. 20 m.
Remain is	70298,106	the sine of 44 40

Second Example.

The sine of 60 deg. is 86602,539 and the co-sine thereof (the sine of 30 deg.) is 50000,000. To find the sines of 59 d. 50 m. and of 60 d. 10 m. here 10 m. is the common difference whose sine before found is 290,887 and co-sine is 99999,577.

1. For the half sum of the sines of 59 deg. 50 min. and 60 deg. 10 min.

As 100000,000 : 99999,577 : : 86602,539 :
Answ. 86602,1727 the half sum of the sines of the extream arches.

2. For the half difference of those sines.

As 100000,000 : 290,887 : : 50000,000 : 145,4435 (the half differ.

Answ. the half sum is	86602,1727	of the sines of 60 deg.
The half difference is	145,4435	10 m. and 59 d. 50 m.
The sum is	86747,6162	the sine of 60 d. 10 m.
The Remainder is	86456,7292	the sine of 59 d. 50 m.

Also putting 30 deg. for the middle arch.
The sine of 29 d. 50 m. is 49747,873.
And the sine of 30 d. 10 m. is 50251,764.

After the same manner may the rest of the sines for every 10 min. be found on each side of 30 d. to 25 d. and to 35 d. And then putting 20 d. for the middle arch, proceed therewith and find the sines on each side thereof, for every 10 m. to 25 d. and 15 d. and so on for 10 d. 40 d. &c.

Furthermore from a due consideration of these two last proportions, for finding the half sum and half difference of the sines of the two extream arches it is evident that this further improvement may be made which will not a little conduce

duce to the facilitating the calculation of the third part of the quadrant, as follows,

And first it is to be noted, That the chord of 60 deg. is always equal to the Radius, *per the 15th. of the 4th. of Euclid.* And consequently the sine of 30 deg. being half the chord of 60, deg. is equal to half the Radius, viz. 50000,00, &c.

In the latter of the two proportions aforesaid, let it be required to produce the 4th. term to be the whole differ. of the sines of the extream arches, then must either one of the middle terms be doubled, or (which is more convenient) make half the Radius the first term, being equal to the sine of 30 deg. (as aforesaid) the co-sine of 60 d.

Therefore if 60 deg. be made the middle arch, by reason of the proportion of equality between the two first terms (i. e. half the Radius and the co-sine of 60 deg.) it consequently follows that the sine of the arch of distance of each of the extream arches from 60 deg. is equal to the difference of the sines of those extream arches. Therefore the sine of one being known the sine of the other is had by Addition or Substraction.

For instance, Let the three arches be 59 d. 50 m. 60 d. and 60 d. 10 m. Then the sine of 10 m. (the arch of distance) is equal to the difference between the sines of 59 d. 50 m. and 60 d. 10. m.

Therefore unto 86456,7292 the sine of 59 d. 50 m.
Add 290,8877 the sine of 10 m.

The sum is 86747,6169 the sine of 60 d. 10 m.

Also (from the same reason) the sine of 20 m. is equal to the difference between the sines 59 d. 40 and 60 d. 20 m. So that the sine of 20 m. added to the sine of 59 d. 40 m. makes the sine of 60 d. 20 m. Likewise the sine of 1 d. is equal to the difference of the sines 59 d. and 61 d. 5 d. is equal to the difference of 55 d. and 65 d. 10 d. is equal to the difference of 50 and 70 d. and so on, always retaining 60 as the middle arch.

Thus far concerning the Construction of the Tables of Natural Sines, to wit, Right Sines and not the Versed Sines, for the versed Sines, as also the Chords aae seldom used in the resolution of Triangles, and therefore Tables thereof are needless and more especially in regard they both may be easily obtained by the right Sines, as follows;

First, Observe that the versed Sine of an arch less then a quadrant with the right Sine complement of the same arch, are together equal to the Radius;

As in Fig. 22. The versed Sine n c with the right Sine of its complement e n (equal T O) is equal to the Radius e c, and therefore the right sine complement of any arch or angle substracted from Radius leaves the versed sine of that arch.

Secondly, The versed sine of an arch greater then a quadrant: is equal to the right sine of the excess of the same arch above a quadrant added to Radius.

As in Fig. 22. The versed Sine a n is equal to the Radius a c added to e n the right sine of the excess (equal T O) and consequently the right Sine of the excess of any arch or angle above a quadrant, added to Radius gives the versed Sine of that arch. Likewise the chords are thus had by the right Sines, the Sine of an arch is (as already defined) half the chord of the arch doubled, and therefore the right sine of any arch or angle doubled gives the chord of the double arch.

As for instance, the Sine of 10 min. doubled makes the chord of 20 min. the Sine of 5 deg. doubled makes the chord of 10 d. the Sine of 20 d. the chord of 40 d. &c.

Note, That in making the Tables of the Natural Sines to great accuracy. The calculation ought always to be continued to 4, 5, 6. or more places then the assigned Radius of the Tables consists of to the end that the fractional part of every division or extraction may be safely neglected without breeding any errour in the following operations.

By

By the Tables of Natural Sines to make the Tables of Natural Tangents and Secants.

HAving the Sines, the Tangents and Secants, may from them be calculated by the following proportions, *viz.*

For the Tangent it holds,

As the co-sine of any arch or angle : Is to the Sine thereof : So is Radius : To the Tangent of that arch.

For the Secant say :

As the co sine of an arch or angle : Is to Radius : : So is Radius : To the Secant of the same arch.

First *Example*, Let it be required to find the Tangent and Secant of the arch of 30 deg. the Sine of 30 deg. is 50000,000 and the co-sine thereof (or the Sine of 60 deg.) is 86602.539. Therefore, To find the Tangent of 30 d. the proportion is,

As	Co-sine 30 d.		Sine 30 d.		Radius
	86602,539	:	50000,000	: :	100000,000

To 57735,028 the Tangent of 30 deg. required.

And to find the Secant of 30 d. the proportion is,

As	Co-sine 30 d.		Radins.		Radius.
	86602539	:	100000,000	: :	100000,000

Answ. 115470,006 the Secant of 30 d. required.

Se-

Second *Example*, Let it be required to find the Tangent and Secant of 60 deg.

First, For the Tangent.

As	Co-sine 60 d.	Sine 60 d.	Radius.
	50000,000 :	86602,539 : :	100000,000

Answ. 173205,078 the Tangent of 60 d. required.

Secondly, For the Secant of 60 d.

As	Co-sine	Radius	Radius
	50000,000	100000,000 : :	100000,000 :

Answ. 200000,000 the Secant of 60 d. required.
The like method observe for any other.

Several more proportions might have been added conducing to the calculation of the Tables aforesaid, but these alone are sufficient for performing the whole Cannon. And such whose genius and curiosity prompts them to a further inquiry into these matters, may have recourse to larger volumes, which have been heretofore mentioned as well for the use of those Numbers in Trigonometry; as also for other ways of their construction, and now it remains to speak something of Logarithms.

IN this place (before we insist upon the Construction of the Tables of Logarithms) it may not be amiss (though a digression) to give a short definition and explication of Progression, so far as is pertinent to our present purpose as being introductory to the subject of Logarithms.

1. Progression (here intended) is twofold, *i. e.* Arithmetical and Geometrical.

2. Arith-

2. Arithmetical Progression or Proportion is either continued or interrupted.

3. Arithmetical Progression continued, or continual proportion Arithmetical: is, when a rank of Numbers do continually either increase or decrease by equal differences, as these ranks or such like.

1, 3, 5, 7, 9, 11, 13, 15, &c. Increasing.

16, 14, 12, 10, 8, 6, 4, 2, &c. Decreasing.

In the first rank there is a continual increase, and in the Second a continual decrease by 2, which is called the common difference or excess.

4. Arithmetical Progression interrupted or discontinued is when four Numbers are proposed having the same difference between the 1st. and 2d. as between the 3d. and 4th. but not retaining the like difference between the 2d. and 3d. as these 2, 4; 8, 10. here 2 and 4, being compared with 8 and 10 do differ by the like excess to wit 2, but so doth not 4 and 8, the like understand of 5, 9, 17, 21, or any other of the like sort.

And this kind may not unfitly (for distinction sake) be called Arithmetical proportion, and the other kind (being a continued series of Numbers equally differing) Arithmetical Progression.

5. In Progressional Numbers whether Arithmetical or Geometrical. The first and the last terms are commonly called the extreams, and the middle terms are called means.

6. Any Three Numbers being proposed in Arithmetical proportion or progression, the double of the mean is equal to the sum of the extreams.

As if the three Numbers be 6, 9, and 12. here the double of 9 (the mean or middle term) is equal to the sum of 6 and 12 (the extreams.) Therefore,

If from the double of the mean, either of the extreams be substracted, there will remain the other.

7. Any four Numbers being proposed in Arithmetical proportion or progression, the sum of the two means is equal to the sum of the two extreams. Thus

Thus 6, 9, 12 and 15 being given, the sum of 9 and 12 (the two means) is equal to the sum of 6 and 15 (the two extreams) the like understand of 13, 17, 21, and 25, or any others of this kind. Therefore, If from the sum of the two means either extream be substracted the remainder is the other.

8. Geometrical Progression or Proportion, is either continued or interrupted.

9. Geometrical Progression continued or continual proportion Geometrical, is when a rank of Numbers, do continually either increase or decrease by the like proportion; that is according to one common multiplyer called the common ratio, as in these or such like, *viz.*

1,	2,	4,	8,	16,	32,	&c. increasing.
3,	6,	12,	24,	48,	96,	&c. increasing.
128,	64,	32,	16,	8,	4,	&c. Decreasing.

In the two first ranks 2 is the common multiplier, and in the last, 2 is a common divisor or (which is the same) $\frac{1}{2}$ is the common multiplier.

10. In a rank of Numbers continually increasing from 1, by Geometrical Progression. The first term after 1, is called the root side, or first power; the second is called the square or second power; the third the Cube or third power; the fourth the fourth power; and so on by the denomination of powers, as the 5th. 6th. 7th. power, &c.

As in the first collumn of the following Table where the root or first power is 2, and in the second collum the root is 3.

11. These powers are produced by a continual multiplication of the root (which is the common multiplier.) Thus, the root squared, produces the square or second power and that multiplied again by the root produces the Cube or third power, and that again by the root produces the fourth power: and so on each power being multiplied by the root produces the power next above it. And the numbers com-

pre

prehended between 1 and the power last produced are called Geometrical mean proportionals, and excluding the first term 1, and the root, all the rest are called a Series, as being form'd from a continual multiplication of the root as aforesaid.

pow.	pow.	exp.
1	1	0
2	3	1
4	9	2
8	27	3
16	81	4
32	243	5
64	729	6
128	2187	7
256	6561	8
512	19683	9
1024	59049	10
&c.	&c.	&c.

12. As the several powers of the roots 2 and 3 in the foregoing Table are form'd by continual multiplication, after the like manner may the powers of any other numbers be form'd, being taken as a root, as 5, 25, 125, 625, &c. or 6, 36, 219, 1296, &c.

13. To these powers it is usual to assign a rank of numbers in Arithmetical progression from unity, 1 being both the first term and common excess, and these are called their exponents or indices, as shewing the distance on seat of each power from unity. As in the third collumn of the preceding Table to 1 is assigned 0, to the root 1, to the second power 2, to the 3d. power 3, &c.

14. The Addition and Substraction of these exponents answers to the multiplication and division of those powers themselves, to which the exponents respectively belong, that is,

15. The sum of the exponents of any two powers is equal to the exponent of that power, which will be produced by the multiplication of the said powers. Thus in the foregoing Table, the sum of 2 (the exponent of the second power) and 3 (the expo. of the 3d power) is 5, which shews, that if the 2d and 3d power be multiplied together they will produce the 5th power. The like understand of any of the rest.

16. Also the difference of the exponents of any two powers, is equal to the exponent of the quotient when the greater of those powers is divided by the lesser, which quotient

ent will be an inferiour power less then the greater of the two former. Thus the difference between 8 and 3 is 5, which implyes that if the 8th power be divided by the 3d the quotient is the 5th power the same is to be understood of any of the rest.

17. Geometrical progresion interrupted or discontinued is when four numbers are proposed, having the like reason or proportion between the first and second, as between the 3d and 4th terms, but not retaining the like reason between the second and third, as in these 3, 6, 8 and 16, here 8 hath the same reason to 16, as 3 hath to 6, (2 being the common multiplier in each) but 6 and 8, the two means) have not the like reason or proportion between them, the same understood 7, 21, 12 and 36, and of all other numbers under the same qualification.

18. And this kind of Geometrical progresion may be rather called Geometrical proportion, and the other kind (which is a series of number continually increasing or decreasing by the like rate or reason) may more properly be termed Geometrical progression. Not but that each are proportional numbers, but one being in a continued progressive order and the other not.

19. Any three numbers being proposed in Geometrical progression or proportion, the square of the mean is equal to the product of the extreams. Thus if three numbers be 9, 12 and 16, the square of 12(the mean) 144, is equal to the product of 16 and 9 (the extreams) the same properties have 6, 12 and 24, &c. Therefore if the square of the mean be divided by either of the extreams, the quotient is the other.

20. Any four number being propounded in Geometrical proportion or progression, the product of the two means is equal to the product of the extreams. Thus 6, 9, 10 and 15 being given the product of 9 and 10 (the two means) is 90, and so is the product of 6 and 15. the same understand of 3, 9, 27 and 81, &c. Therefore if the product of the two means be divided by either of the extreams the quotient is the other.

21. And from hence ariseth that excellent Rule in Arith-

metick

metick called the Golden Rule or Rule of Three, in all the operations whereof three proportional numbers are known, and thereby a fourth unknow is found out.

23. The numbers of Multiplication and Division are proportionals. Thus in Multiplication it holds As 1, is to one of the Factors, So is the other: to the product.

And in Division it holds. As the Divisor is to 1, so is the dividend to the quotient. Or as the Divisor is to the Dividend, So is 1 to the quotient.

24. Four proportional numbers are usually distinguished with points thus 6 : 9 : : 10 : 15, &c.

25. Furthermore. As in the 14, 15, and 16 foregoing Addition of the exponents answers to the multiplication of their powers, and Substraction to Division. The same holds if to any other rank of numbers in Geometrical progression, be assigned any rank of numbers in Arithmetical progression, viz. the Addition and Substraction of the latter answers to Multiplication and Division of the former, from which notion the Logarithms takes their rise; which was the cause of prefixing this discourse of progression by way of preface to the Logarithms; being so much thereof as seems necessary, for their better illustration. And as to such Arithmetical questions as usually belong to Arithmetical and Geometrical progression, they are on purpose here omitted, as being less material to the Subject of Logarithms, and because of designed brevity. I shall refer for such cheifly to Dr. *Wallis's Opus Arithmeticum*; or for want thereof to *Moores Arithmetick*, *Wingates Arithmetick* by *Kersey*, and divers others; So with thus much conclude both kinds of Progression.

Of

Of LOGARITHMS.

First, *Of the Rise and Nature of Logarithms.*

AS the first Famous Inventer of the Logarithms the Honourable Lord *Neper* of *Scotland*, together with the Learned and Laborious Mr. *Henry Brigs*, who afterwards (by the joint consent of the Author) reduced them into a better form and compleated them. I say as to these Worthies though much is due yet little need be said of their praise, which has long since (to the great credit of them and their Nations) been spread through all or most parts of *Europe*, and many other parts of the World where Science is promoted: Their labours will continue their fame to posterity, nor can their Names and memory ever dye, as long as those their monuments the Logarithms remains in being: Nor need I here insist upon the usefulness of those numbers which is so general in all the parts of the Mathematicks, and especially in regard so many Treatises have already been published in our own Language, and therein their uses very largely exemplified. What is here intended concerning Logarithms is something of their Original and Nature, but cheifly their Construction or making.

St-

Secondly, *Of the Original and Nature of Logarithms.*

Definition. LOgarithms are borrowed Numbers in Arithmetical Progression, fitted or assigned to a rank of Numbers in Geometrical progression.' Therefore, any rank of Numbers being given in Geometrical progression to them may be annexed for Logarithms, any rank of Numbers in Arithmetical progression at pleasure.

As in this Table, in the first column thereof, is a rank of numbers in Geometrical progression from 1. Now to those or to any other rank of proportionals may be adjoined for Logarithms either of those ranks in the 2d. and 3 d.col. or any other rank of equi-different numbers, but those are most commodious for use which have o assigned for the Logarithms of Unity.

Num.	Log.	Log.
1	0	0
2	1	3
4	2	6
8	3	9
16	4	12
32	5	15
64	6	18
128	7	21
256	8	24
512	9	27
&c.	&c.	&c.

These Logarithms or Artificial numbers being thus contrived and assigned to proportional numbers and respectively substituted instead thereof; those conclusions which in the proportional numbers are wrought by Multiplication and Division, may be performed by the Addition and Substraction of their correspondent Logarithms; and the extraction of roots in the proportionals may be ef-

fected by Division and the production of powers by Multiplication of their correspodent Logarithms; for from the aforesaid Definition and explication of Logarithms these consequences follow.

1. Of four numbers in Geometrical proportion, The Logarithm of the first, being substracted from the sum of the Logarithms of the 2d. and 3d. the remainder is the Logarithm of the fourth.

This is clear from the 7th. preceding of progression Arithmetical.

Example.

Let there be given three proportional numbers as 4, 16 and 32. To find the fourth,

The Proportion is,

	Log.
As 4	2
To 16	4
So is 32	5
	9
To 128	7

Here the sum of 4 and 5 (the Logarithms of the 2d. and 3d.) is 9 from which abating 2 (the Logarithm of the 1st.) there remains 7 the Log. of (128) the 4th. number which was required.

2. In Multiplication the sum of the Logarithms of the Factors is equal to the Logarithms of the product. This is evident from the 23d. foregoing of Geometrical progression.

Example.

Let the Factors be 32 and 8, to be multiplied together.

The

The Proportion is,

	Log.
As 1	0
To 8	3
So is 32	5
To 256	8

Here because the Logarithm of 1 (the 1st. number) is 0, therefore 8 the sum of 3 and 5 (the Logarithms of the 2d. and 3d.) is the Logarithm of 256 the product.

3. In Division. The Difference of the Logarithms of the dividend and divisor is equal to the Logarithm of the quotient:

This is also manifest from the 23d. of Geometrical progression aforesaid.

Example.

Let 256 be devided by 8.

The Proportion is,

	Log.
As 8	3
To 1	0
So is 256	8
To 32	5

Here because the Logarithm of 1 (the 2d. number) is 0, therefore 3 (the Logarithm of the 1st. or divisor) substracted from 8 (the Logarithm of the 3d. or dividend) leaves 5, the Log. of 32 the quotient.

4. In the production of powers of numbers. The Logarithm of the root multiplied by the exponent of the power gives the Log. of the same power.

This is obvious from the 14th. and 15th. of Geometrical Progression.

Example.

Let 4 be the root given, to find the Square, Cube 4th. power, &c. thereof.

If 4 be put for the root then its Exponent is 1, and consequently the 2, 3, 4. &c. are the exponents of the other powers thereof in their respective order. Therefore

In the 1st. column of the preceding Table of Logarithms.

The Log. of 4 is 2
Which mult. by 2 the exp. of the Squ. or 2d. power.

produces 4 the Log. of 16 the squ. or 2d. power.

Again 2 the Log. of the root.
Mult. by 3 the exp. of the Cube or 3d. power.

produces 6 the Log. of 64 the Cube, or 3d. power of 4.

The like may be performed for any other power of 4, or for the powers of any other number.

5. In the Extraction of Roots. The Log. of the power divided by its exponent quotes the Logarithm of the root respectively.

This is Converse of the former, and obvious from the 16th. of progression Geometrical.

Example.

Let 64 be given to find the square or Cube root thereof. Here 6 (the Log. of 64) divided by 2 (the exp. of the 2d. power) quotes 3 the Log. of 8, and therefore 8 is the squ. root of 64. Also 6 divided by 3 (the expon. of the 3d. power) quotes 2 the Log. of 4. Therefore 4 is the Cube-root of 64: the like method observe, for finding the roots of any

other

other power, or numbers commenſurable by their roots, and thoſe preceeding concluſions or operations which are performed by the Logarithmical numbers in the 2d. column of the preceding Table, may alſo be performed by thoſe in the 3d. column, or by any other rank of numbers in Arithmetical progreſſion.

And as thoſe numbers either in the 2d. or 3d. column of the Table aforeſaid, may be aſſigned for Logarithms to the rank of proportionals in the firſt column. In the like manner, may thoſe (or any other of the like ſort) be aſſigned for Log. to any other rank of proportionals whatſoever.

Note alſo, That though the preceding operations in diviſion and extraction of Roots are performed only in ſuch numbers as are commenſurable by their diviſors and roots. Yet the ſame holds and may be effected in Numbers that are incommenſurable by the help of a large Table of Logarithms, *viz.* Such as are in *Briggs* his *Logarithmica Arithmetica*, or in *Gillibrands* or *Newtons Trigonometria Britanica*, and divers others.

So judging thus much ſufficient to illuſtrate the foundation and nature of Logarithms we will now proceed to ſpeak of their Conſtruction or making, as they are in the Tables now generally uſed.

Thirdly, *Of the Conſtruction of Logarithms.*

ALthough (or hath been already ſhown) ſeveral kinds of Logarithms might be aſsigned for the ſame proportional numbers, or ſeveral proportional numbers to the ſame Logarithms, yet thoſe are found moſt commodious and apt

for the use in all Arithmetical operations, which (having a Cypher for the Logarithm of unity) are fitted to a rank of numbers in Geometrical progression increasing from unity by a decuple proportion, as thus, 1 with competent number of cyphers annexed is the Log. of 10; 2 with the like number of cyphers for the Log. 100, and so on as in the following Table.

Numbers.	Logarithms.
1	0,00000
10	1,00000
100	2,00000
1000	3,00000
10000	4,00000
100000	5,00000
1000000	6,00000

In the first column is a rank of proportional numbers increasing from 1. in a Ten-fold reason: In the second column is their respective Logarithms wherein 1 or (rather) 1,00000 is their common difference.

To these proportinal numbers, *viz.* 1, 10, 100, &c. in the first colum are assigned for Logarithms the numbers 1, 2, 3. &c. in the 2d. column with as many Cyphers annexed; as are the number of places designed for the Table of Logarithms to consist of, and those Cyphers may (not improperly) be esteemed as decimal places. So that the Logarithms hereby becomes mixt numbers whose integers (which is always the first figure in each towards the left hand) being separated from the decimals by a point, are called the Characteristicks or Indices of the Logarithms, because they shew the distance or seat of their correspondent numbers from the units place, as being always less by an unit then the number of figures or places the natural or absolute number, belonging thereto

doth

doth consist of, thus the index of the Log. of all numbers under 10 is 0. The index of the Log. of all numbers between 10 and 100 is 1, of all between 100, and 1000 is 2, &c. The Logarithms being thus assumed to be mixt numbers, it follows, that the Logarithms of all numbers between 1 and 10 begin from 0. and are consequently less then an unit. The Log. of all between 10 and 100 begin from 1 and are therefore greater then 1 but less then 2 and so on in the like order successively.

Having thus assigned the Log. of 1, 10, 100, &c. to be 0,00000, 1,00000, 2,00000, &c. as in the preceding Table, by those to find the Logarithms of all the inclusive integral numbers between 1 and 10, as 2, 3, 4, 5, 6, 7 8, 9, those between 10 and 100, as 11, 12, 13, 14, 15, &c. is the subject of the ensuing part.

The old way of making them was (by the first inventors thereof before-mentioned) laid down by the extraction of many roots, thereby to find so many continual mean proportionals until the number of cyphers intercepted between unity, and the first significant figure of the fractional part, was equal to the number of places that the intended Table of Logarithms should consist of. And to those proportionals so many Arithmetical means was to be found, for Logarithms respectively.

Another method was by continual Multiplication, which Dr. *Newton* prosecutes in his *Trigonometria Britanica*, deduced (as he intimates in his Preface) from the 5th. Chapter of *Briggs Arithmetica Logarithmica*, but both these ways was very laborious as well as tedious; which Considerations put many ingenious men and able Mathematicians, upon an inquiry and diligent search for finding out a more expeditious way, then what was then known: Whereupon one Mr. *James Gregory* of *Scotland* in an excellent small Treatise of his called, *Vera Circuli & Hyperbolæ quadratura*, Printed at *Padua* in the year, 16 doth shew how (with great accuracy) to produce Logarithmical numbers (by squaring the Geometrical figure called the *Hyperbola*) to any number of place desired

fired, and that with more ease and speed, then by any of the methods before known, and these numbers he calls hyperbolical Logarithms. The same or like method was pursued by Mr. *Nicholas Mercator*, in a small (but Learned) Treatise of his Intituled *Logarithmotechnia*, Printed at *London* in the year 1668. and by some new properties which he then discovered in the *Hyperbola*, did from thence deduce the said hyperbolical Logarithms with more ease and expedition then by *Gregory*'s method, after that in the same year 1668. The said *James Gregory* reassumed the same subject, and made further improvement and demonstration thereof according to *Mercators* method, and this in an ingenious Treatise of *Gregory*'s published by the Title of *Exercitationes Geometricæ*, which method is now generally accounted the best; also a further Explanation of *Gregory*'s method (in his *Exercitationes Geometricæ*) hath been lately published in English by Mr. *Enal. Speidal* (all the former of *Gregory's* and *Mercator* being in Latin) with a Geometrical figure for demonstration, of affinity with the *Hyperbola*. The same method (as deduced from the aforesaid Treatise *Exercitationes Geometricæ*) I shall here insist upon referring the more inquisitive or intelligent Reader to the said Treatise for Geometrical demonstration thereof; and contenting my self with the Numerical or Figurative part of the work in this small Treatise, shall wave all further discourse hereof, and only endeavour to lay down the Rules, and explain the operations thereof in as plain and intelligible terms as possibly I can, considering the Subject.

To make the Hyperbolical Logarithm of any Number, the General Rule may be briefly laid down thus;

TO the given Number add 1 for a denominator or divisor, and from the given number Substract 1 for a Numerator or dividend; then of the vulgar fraction hence result-

resulting compose all the odd powers thereof, which will be a series of proportional numbers, and divide those powers by their respective Exponents, to wit, 1, 3, 5, 7. &c. The sum of all those quotes (being reduced first into their least terms and then into Decimals) is half the Hyperbolick Logarithm of the number proposed, and from those Hyperb. Log. thus found, may the Tabular Log. now generally used, be easily obtained, as shall hereafter be shown in its due place.

First Example.

Let it be required to make the Hyperbolick Logarithm of 2, according to the General Rule. To the given number 2, I add 1 the sum is 3 for a denominator or divisor; and from 2 substracting 1, leaves 1 for a numerator or dividend, then 1 divided by 3 is $\frac{1}{3}$, so that the vulgar fraction hence resulting is $\frac{1}{3}$, and the several odd powers of $\frac{1}{3}$ are as by their odd Exponent thus,

Expon.	1,	3,	5,	7,	9,	&c.
Powers.	$\frac{1}{3}$,	$\frac{1}{27}$.	$\frac{1}{243}$,	$\frac{1}{2187}$,	$\frac{1}{19683}$,	&c.

That those powers are proportionals is evident from the 9th. of *Geom. Progression* foregoing, in that they have the same common ratio or multiplier, *i. e.* $\frac{1}{9}$ the square or 2d. power of $\frac{1}{3}$, and by the 15. of *Geom. Prog.* its apparent that all those odd powers may be formed by a continual multiplication of $\frac{1}{3}$ by $\frac{1}{9}$ its square or 2d. power, Thus,

$$\left.\begin{array}{ll}\text{Expon.} & 2 \times 1 = 3 \\ \text{Powers} & \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}\end{array}\right\}\text{and}\left\{\begin{array}{l}3 \times 2 = 5, \&c. \\ \frac{1}{27} \times \frac{1}{9} = \frac{1}{243},\end{array}\right.$$

Those Powers being divided by their respective Indices or Exponents, which according to division in vulgar Fractions, is

is only multiplying the several denominators, by their corresponding exponents respectively thus,

$\frac{1}{3} \times 1 = \frac{1}{3}$, $\frac{1}{27} \times 3 = \frac{1}{81}$, and $\frac{1}{243} \times 5 = \frac{1}{1215}$, &c.

which done, the several quotients, are,

$\frac{1}{3}, \frac{1}{81}, \frac{1}{1215}, \frac{1}{15309}, \frac{1}{177147}, \frac{1}{1948617}, \frac{1}{20726199}, \frac{1}{215233605}$, &c.

These quotes being Reduced into Decimals, are

$$
\begin{array}{rcl}
\frac{1}{3} & = & ,333333333,\ \&c. \\
\frac{1}{81} & = & ,012345679, \\
\frac{1}{1215} & = & ,000823045 \\
\frac{1}{15309} & = & ,000065321 \\
\frac{1}{177147} & = & ,000005645 \\
\frac{1}{1948617} & = & ,000000513 \\
\frac{1}{20726199} & = & ,000000048 \\
\frac{1}{215233605} & = & ,000000005 \\
\hline
\frac{1}{2}\ \text{the hyp. Log. of } 2 & = & ,346573589 \\
 & & 2 \\
\hline
\text{The Hyp. Log. of 2 is} & = & ,693147178 \\
\end{array}
$$

By the Log. of 2 may the Logarithms of all the powers of 2 be easily made, *viz.* The Log. of 4, 8, 16, 32, &c. but this I shall refer till after I have shown how by the Hyperbolick Log. of 2. To find the Tabular (or *Briggs's*) Logarithm of 2.

And here Note, That though this method for Calculating the hyperbolick Logarithms, by the general Rule aforesaid, be

be both easie certain, and expeditious: and may therefore be used for all numbers whatsoever, yet there may be some Abbreviations or Compendiums deduced, which will be of good advantage in the operations and these Abbreviations, are;

First, Instead of framing the powers aforesaid in the terms of vulgar Fractions, they may be produced in Decimals thus.. The Decimal of $\frac{1}{3}$ (the first power in the preceding Example) is ,333333333, &c. but to produce odd powers of ,333333333, &c. = $\frac{1}{3}$ by a continual Multiplication of ,111111111, &c. the Decimal of $\frac{1}{9}$ the 2d. power of $\frac{1}{3}$, would be intollerable, yet as $\frac{1}{9}$ or its Decimal is here a continual multiplicator, from thence it follows (and is apparent from the foregoing Treatise of Decimal Arithmetick, in *page* 86, 87, and 88. Where is shown how by a given divisor to find a multiplicator and the contrary) that a divisor may thereby be had which will effect the same, as the continual multiplicator aforesaid; *viz.* As $\frac{1}{9}$: 1 :: 1 $\frac{9}{1}$ or 9 the divisor required. So that as $\frac{1}{9}$ is here a continual multiplicator. So may 9 be a continual divisor; Therefore divide ,333333333 by 9 continually; that is, ,333333, &c. divided by 9 quotes ,037037037, and that quotient again by 9 quotes ,004115226 and so on, setting the quotients underneath each other, which quotients are the odd powers of $\frac{1}{3}$ in Decimals, and consequently are proportionals as before; which done divide those powers by their respective Exponents. The sum of their quotients thus found shall make half the hyperbolick Logarithms of 2. agreeing with the other method. And

The

The Operations in Decimals are accordingly as follows, for making the Hyperbolick Logarithm of 2 $,\frac{1}{3}$ = 3333, &c.

The several odd powers of ,333333333 = $\frac{1}{3}$ form'd by dividing by 9 continually, are		Those Powers being divided by their correspond. expo: the quotes to be added for $\frac{1}{2}$ the Hyp. Log. of 2 are
Expon.	Powers.	
1	,333333333,&c.	,333333333
3	,037037037	,012345679
5	,004115226	,000823045
7	,000457247	,000065321
9	,000050805	,000005643
11	,000005645	,000000513
13	,000000627	,000000048
15	,000000069	,000000005

Half the Hyperb. Log. of 2 is ,346573589
2

The Hyp. Logarithm of 2 is ,693147178 as before.

Hence you may chuse whether you will perform the Operations in terms of vulgar Fractions, and reduce the last quotients into Decimals, or effect the whole Operations in Decimals. I have here for better illustration, inserted the Operations according to each method for making the Hyperb. Logarithm of 2, but in the following Examples shall perform the Operations in Decimals according to the latter method. And this explication I judge sufficient, in which I have been the more large to the end that these precepts may serve in all the ensuing Examples or for making the hyp. Log. of any number without further directions.

Second Example.

Let it be required to make the Hyperbolick Logarithm of 3. First,

First, $3 - 1 = 2$, for the numerator or dividend and $3 + 1 = 4$, for the denominator or divisor therefore the vulgar Fraction resulting is $\frac{2}{4} = \frac{1}{2}$ the square of $\frac{1}{2}$ is $\frac{1}{4}$ for a continual multiplicator and by consequence 4 is a continual divisor, $\frac{1}{2}$ is (in Decimals) ,50000000, &c.

The Operations in Decimals are as follows.

The odd powers of ,5 = $\frac{1}{2}$ produced by a continual division by 4.		The quot. (of the several pow. of ,5 being divided by their corresponding Expon.) to be added to make $\frac{1}{2}$ of the Hyperb. Log. of 3.
Expon.	Powers.	
1	,50000000, &c.	,50000000
3	,12500000	,04166667
5	,03125000	,00625000
7	,00781250	,00111607
9	,00195312	,00021701
11	,00048828	,00004439
13	,00012207	,00000939
15	,00003052	,00000203
17	,00000763	,00000045
19	,00000191	,00000010

Half the Hyperb. Log. of 3 ,54930611
The Hyperb. Log. of 3 is 1,09861222

Another Abbreviation or Compendium in making the hyperbolical Logarithms may be this.

That although the hyperb. Log. of any number may be made by the Addition and Substraction of 1 to and from the number proposed, &c. (according to the preceding General Rule). Yet when the given number is such, as that the vulgar Fraction resulting (by adding and Substracting 1 &c.) hath not 1 for its Numerator. Then the Log. of such a number is more readily made by the Logarithms of its two Composers or Factors whose Fact or Product is equal to the given number. And one of the Factors ought to be such a num-

number whose Log. is already known. The other such a number, as that by the Addition and Substraction of 1, there may result a vulgar Fraction having 1 for its numerator, so may the several powers thereof be more easily formed, and consequently all the operations more expedited and facilitated, then when the Numerator is any number greater then 1;

Thus if you desire to make the hyp. Log. of 3 by two numbers whose Fact is 3, the two numbers most convenient will be 2 and $1\frac{1}{2}$, for the Logarithm of 2 is known, and 1 added to, and substracted from $1\frac{1}{2}$, leaves the result of $\frac{1}{5}$ the square whereof is $\frac{1}{25}$ for a continual Multiplicator whereby to form the odd powers thereof, and consequently 25 is a continual divisor to effect the same, or rather as ,04 is the equivalent Decimal of $\frac{1}{25}$, ,04 may therefore be made a continual multiplicator for 'tis all one, whether you divide by 25, or multiply by $\frac{1}{25}$, or by ,04, continually for the production of the odd powers of $\frac{1}{5}$ or ,2 its equivalent *Decimal*.

The Operations (to make the hyp. Log. of $1\frac{1}{2}$) are in Decimals as follows;

The odd powers of ,2 = $\frac{1}{5}$ produced by continual dividing by 25, or (which is the same) by continual multiplying by ,04.		The several quotes (of the powers of ,2 divided by their proper Expon.) to be added to make $\frac{1}{2}$ the hyper. Log. of $1\frac{1}{2}$ are.
Expon.	Powers.	
1	,200000000,&c	,200000000, &c.
3	,008000000	,002666667
5	,000320000	,000064000
7	,000012800	,000001828
9	,000000512	,000000057
11	,000000021	,000000002

Half the hyp. Log. of $1\frac{1}{2}$ is = ,202732554

The

The hyp. Log. of $1\frac{1}{2}$ is $=$,405465108.
The hyp. Log. of 2 added $=$,693147178

The sum is the hyp. Log. of $3 =$ 1,098612286 nearly agreeing with the hyp. Log. of 3 before found, and those two ways may serve to verifie or prove each other.

Note; That because $2 \times 1\frac{1}{2} = 3$, therefore the Log. of 2 added to the Log. of $1\frac{1}{2}$ makes the Log. of 3.

By the Log. of 3 may the Logarithms of all the powers of 3, *viz.* 9, 27, 81, &c. be easily obtained. Likewise having the hyp. Logar. of 2 and 3 the hyp. Log. of 6 (their Fact) is readily had, for because $2 \times 3 = 6$. Therefore

To the hyp. Log. of $3 =$ 1,098612286
Add the hyp. Log. of $2 =$,693147178

Sum is the hyp. Log. of $6 =$ 1,791759464

The Third Example. To make the Hyper. Log. of 5.

To make the hyp. Log. of 5. The best and most facile way is to chuse two numbers (according to the preceding directions) whose Fact is 5, and those numbers may be either 3 and $1\frac{2}{3}$, or 4 and $1\frac{1}{4}$.

First, by the hyperbolick Logarithms of 3 and $1\frac{2}{3}$. To make the hyperbolick Logarithm of 5, The hyperbolick Logarithm of 3 is already known. Therefore the hyp. Log. of $1\frac{2}{3}$ is required to be made.

$1\frac{2}{3} - 1 = \frac{2}{3}$ for the numerator or dividend, and

$1\frac{2}{3} \times 1 = 2\frac{2}{3}$ or $\frac{8}{3}$ for the denominator or divisor, and

$\frac{2}{3}$ divided by $\frac{8}{3}$ (rejecting the nenominators in each) is $\frac{2}{8}$ or $\frac{1}{4}$. The square of $\frac{1}{4}$ is $\frac{1}{16}$, the continual multiplicator and therefore 16 is a continual divisor, whereby to produce the odd powers $\frac{1}{4}$ or ,25 the decimal equivalent to $\frac{1}{4}$.

The

The Operations are

The odd powers of ,25 = $\frac{1}{4}$ produced by dividing continually by 16.		Those several powers being divided by their proper Expon. respectively. The quotes to be added are
Expon.	Powers.	
1	,250000000,&c	,250000000
3	,015625000	,005208333
5	,000976563	,000195313
7	,000061035	,000008719
9	,000003815	,000000424
11	,000000238	,000000022
	,000000015	,000000001

Half the hyperb. Log. of 1 $\frac{2}{3}$ is = ,255412812

The hyperb. Log. of 1 $\frac{2}{3}$ is = ,510825624
The hyperb. Log. of 3 added = 1,098612286

The Sum is the hyp. Log. of 5 = 1,609437910
To which add the hyp. Log. of 2 = ,693147187

The Sum is the hyp. Log. of 10 = 2,302585097

Here the Log. of 3 is added to the Log. of 1 $\frac{2}{3}$ to make the Log. of 5, because 3 × 1 $\frac{2}{3}$ = 5. Also because 5 × 2 = 10, therefore the Log. of 2 added to the Log. of 5 gives the Log. of 10;

By the Log. of 5 may the Logarithms of the powers of 5 be readily made as of 25, 125, 625, &c.

Secondly, To make the hyp. Log. of 5, by its Factors 4 and 1 $\frac{1}{4}$, the hyp. Log. of 4 is already known (being twice the Log of 2 for that 2 × 2 = 4) so that it is only here required to make the hyp. Log. of 1 $\frac{1}{4}$.

1 $\frac{1}{4}$ — 1 = $\frac{1}{4}$

$1\frac{1}{4} - 1 = \frac{1}{4}$ for a Numerator or dividend, and

$1\frac{1}{4} + 1 = 2\frac{1}{4}$ or $\frac{9}{4}$ for the denominator or divisor. So that the vulgar fraction hence resulting is $\frac{1}{9}$ whose square is $\frac{1}{81}$ for a continual Multiplicator, and therefore 81 is a continual Divisor, whereby to raise the odd powers of $\frac{1}{9}$ or ,111111111, &c, the decimal corresponding thereto.

The Operations are

The odd pow. of ,111111111 &c. = $\frac{1}{9}$ produced by dividing continually by 81, or (which is all one) by dividing twice by 9.		The quotes to be added to make $\frac{1}{2}$ hyp.log.of $1\frac{1}{4}$, being the odd powers of $\frac{1}{9}$ divided by their corresponding Exp. respectively.
Expon.	Powers.	
1	,111111111,&c	,111111111,&c.
3	,001371742	,000457247
5	,000016935	,000003387
7	,000000209	,000000030
9	,000000003	

Half the hyperb. Log. of $1\frac{1}{4}$ is = ,111571775

The hyperb. Log. of $1\frac{1}{4}$ is = ,223143550

hyp. Log. of 2 beforefound is = ,693147178

The hyperb. Logarith. of 4 is = 1,386294356
To which add the hyp.Log.of $1\frac{1}{4}$ = ,223143550

The Sum is the hyp. Log. of 5 = 1,609437906

Note that if the work had been continued to more places those two ways of calculating the hyperb. log. of 5 would have more nearly agreed, and may serve to verifie each other.

Here Note also, That although the readiest method to make the hyperb. Log. of 5 be (as aforesaid) by chusing two numbers whose fact is 5, which may be either 3 and 1 $\frac{2}{3}$ or 4 and 1 $\frac{1}{4}$ (as in the two preceding operations) and adding their logarithms together. Yet the hyp. log. of 5 may (with some more difficulty) be made by adding and substracting to and from 5 the given number it self, according to the first general Rule, but in regard the operations are too large for this narrow page, and because the hyperbolick logarithm of 5 is already done two wayes. I shall omit the making thereof by the number it self, and leave that to the practise of the ingenious that shall think fit to perform it that way.

The Fourth Example.

To make the Hyperbolick Logarithm of 7.

The Hyperbolick logarithm of 7 may with much facility and dispatch be made by its factors, to wit, either by 5 and 1 $\frac{2}{5}$ or 6 and 1 $\frac{1}{6}$. In making the hyperbolick logarithm of 7 by the two first numbers, the hyperbolick logarithm of 5 being already known, there is only the hyperbolick log. of 1 $\frac{2}{5}$ required to be made in order to obtain the hyperb. log. of 7.

$1\frac{2}{5} - 1 = \frac{2}{5}$ the numerator or dividend. Also

$1\frac{2}{5} + 1 = 2\frac{2}{5}$ or $\frac{12}{5}$ the denominator or divisor, therefore the fraction resulting hereof is $\frac{2}{12}$ or $\frac{1}{6}$ the square of which is $\frac{1}{36}$ for a continual Multiplicator or 36 for a continual divisor, whereby to produce the odd powers of $\frac{1}{6}$ or ,16666666, &c. its corresponding decimal.

The

The Operations are.

The odd pow. of ,16666667 = $\frac{1}{6}$ produced by continual division by 36.		Those powers being divided by their respective Expon. the quotes to be added (for half the hyp.log.of 1 $\frac{2}{5}$) are
Expon.	Powers.	
1	,1666666666&c	,16666667
3	,0046296296	,001543210
5	,0001286008	,000025720
7	,0000035723	,000000510
9	,0000000992	,000000011
11	,0000000028	

$\frac{1}{2}$ the hyp. log. of 1 $\frac{2}{5}$ is = ,168236118

The hyp. log. of 1 $\frac{2}{5}$ is = ,336472236
Unto which add the hyp.log.of 5 = 1,609437910

The sum is the hyp.log.of 7 = 1,945910146

Because the fact of 5 and 1 $\frac{2}{5}$ is 7, therefore is the hyp. log. of 5 added to the hyperb. log. of 1 $\frac{2}{5}$ to obtain the hyp. loga. of 7.

To make the hyp. log. of 7 by its two factors 6 and 1 $\frac{1}{6}$ the hyp. log. of 6 is known, and the fraction resulting by adding and subducting 1 to and from 1 $\frac{1}{6}$ is $\frac{1}{13}$ whose squ. $\frac{1}{169}$ for a continual Multiplicator, and 169 is therefore the continual divisor for the production of the odd powers of $\frac{1}{13}$ = ,0769230769, &c.

The Operations are

Odd pow. of ,0769230769, &c. $= \frac{1}{13}$ produced by dividing continually by 169, or by dividing twice by 13.		The powers being divided by their proper Expon. The quotes to be added to make ½ the hyp. log. of 1 $\frac{1}{6}$ are
Expon.	Powers.	
1	,0769230769&c	,076923077
3	,0004551662	,000151722
5	,0000026933	,0000,0539
7	,0000000159	,000000002
9	,0000000001	

Half the hyperb. log. of 1 $\frac{1}{6}$ is = ,077075340

The hyp. log. of 1 $\frac{1}{6}$ is = ,154150680
The hyp.log.of 6 being added = 1,791759464

The Sum is the hyp. log. of 7 = 1,945910144 nearly agreeing with the hyp. log. of 7 before found by the factors 5 and 1 $\frac{2}{5}$.

By the log. of 7 the log. of 49, 343, 2401, &c. the powers of 7 is easily made by multiplication only. Having made the hyp. log. for the numbers 3, 5, and 7 by two methods, for this end that one many serve to verifie or prove the other, it may be convenient to give a hint how to make the hyp. log. of 2 by two factors to prove the foregoing operation of making the hyp. log. of 2 by the number it self.

Thus the hyp. log. of 2 may be easily and readily made by the numbers 1 ½ and 1 ⅓ whose fact is 2. And in order hereunto the hyp. log. of 1 ½ has been already made therefore hyp. log. 1 ⅓ is required which by operation will be found to be ,287682072
To which add the hyp. log. of 1 ½ = ,405465108

The sum is the hyperb. log. of 2 = ,693147180 nearly agreeing with the hyp.log. of 2 before found.

These

These Precepts and Examples I judge sufficient, for making those numbers called hyperbolick Logarithms. The like method being to be observed for the hyperbolick Logarithms of all incomposite Numbers, *i. e.* Such Numbers as are not commensurable by any whole number between an unit and the number it self, that is 11, 13, 17, 19, 23, 29, &c.

Thus the hyperbolick Logarithm of 11 may easily be made by its two Factors 10 and $1\frac{1}{10}$, the Logarithm of 10 being known, and in order to make the hyperbolick Logarithm of $1\frac{1}{10}$ the fraction resulting will be $\frac{1}{21}$.

Or the hyperbolick Logarithm of 11 may be readily made by the two Factors 9 and $1\frac{2}{9}$, their fact being 11 and the fraction resulting from $1\frac{2}{9}$ is $\frac{1}{10}$.

Also the hyperbolick Logarithm of 13 may be made either by its two Factors 12 and $\frac{1}{12}$ (the fraction resulting from $1\frac{1}{12}$ is $\frac{1}{25}$) or by its two factors 11 and $1\frac{2}{11}$ the fraction resulting from $1\frac{2}{11}$ is $\frac{1}{12}$. The like Compendious method may be performed for the other incomposite numbers, by two factors, which I leave to the search of the ingenious Practitioner herein.

By the hyperbolick Logarithms of those numbers already made. The Tabular Logarithms of all numbers under 10 is easily obtained, and from thence likewise is readily had the Tabular Logarithms of all composite numbers, arising from the multiplication of any two or more of those numbers together, whose Logarithms is known as follows.

For as much as all Logarithms performs the same conclusions, therefore they are proportional, that is, As the hyperbolick Logarithm of any number: Is to the Tabular (or *Briggs*) Logarithm of that number : : So is the hyperbolick Logarithm of any other number : To the Tabular logarithm of the same number. But the Tabular logarithm of 10 is always 1,0000000, &c. (as hath been already shown) and the hyperbolick Logarithm of 10 is (as before found) 2,302585088. Therefore the proportion may (universally and most conveniently) run thus. As the hyperbolick logarithm of 10 : is to the Tabular Logarithm of 10 : : So is

is the hyperbolick Logarithm of 2, 3, 4, 5, or any other number : To the Tabular Logarithm of the same number.

And here in as much as the two first terms may be general, they may therefore be accepted of as a fixt ratio between the hyperbolick and the tabular Logarithms, and from hence it follows (according to the method delivered in page 89 and 90, in the preceding Treatise of Decimal Arithmetick, for finding a fixt Multiplicator or Divisor in all such Cases where there is a determined ratio) that the fixt Divisor may be $\frac{2302585088}{1000000000}$ or rather 2,302585, &c. And the fixt multiplicator may be $\frac{1000000000}{2302585088}$ or rather ,0434294483, &c. found by dividing 1,000000000, &c. (the Tabular Logarithm of 10) by 2,302585088 (the hyperbolick logarithm of 10) as follows.

2,302585088) 1,0000000000 (,0434294483, &c.

7896596480

9888412160

6780718080

21755479040

10322142480

11117921280

19073809280

653128576, &c.

Hence it is that if the hyperbolick logarithm of any number be either divided by 2,302585, or multiplied by ,04342945, the quotient or product is the tabular logarithm of that number. Thus the hyperbolick logarithm of 2 is ,69314718o, the tabular Logarithm by division is ,30103000, before

before which prefixing 0 for its proper Index or characterstick and then it is 0,30103000, the logarithm of 2 according to *Brigg*'s Tables.

2,302585) ,693147180 (,301030007, &c.

2371680

6909500

17450000, &c.

By Multiplication the tabular Logarithm of 2 is the same (*prope verum*) as by Division, as follows.

The hyperbolick Logarithm of 2 is = ,693147180
The fixt Multiplicator is = ,4342945

3465735900
2772588720
6238324620
1386294360
2772588720
2079441540
2772588720

The Tabular Logarith. of 2 is 0,30103000,79645100

For the Logarithms of the powers of 2, *viz.* 4, 16, 32, 64, &c. Multiply the Logarithms of the root 2, by the Exponents of the powers as 2, 3, 4, 5. &c. the several products are the Logarithms of the powers of 2, respectively. Thus 0,30103000 the tab. log. of 2 multiplyed by 2 gives 0,60206000 the tab. log. of 4. And by 3 gives 0,903900, the tab. log. of 8, and by 4 gives 1,2041200 the tab, log. of 16, and so on for the rest of the powers of 2, *ad infinitum.*

The like is to be performed by the logarithms of any other number to find the logarithms of the powers of the same number;

By

By the hyperbolick Logarithm of 3. To find the Tabular Logarithm of 3.

The hyperb. Log. of 3 (before found) is = 1,098612286
The fixt Multiplicator is = 04342945

5493051430
4394449144
9887510574
2197224572
4394449144
3295836858
4394449144

The Tabular Log. of 3 is = 0,477121212,734412270

The Tabular Log. of 9 is = 0,9542425
of 27 is = 1,4313638
of 81 is = 1,9084850

By the Logarithms of 2 and 3 the Logarithms of all their Composites, as 6, 12, 18, 24, 36, &c. is readily obtained by Addition alone. For the Logarithm of 2 added to the Log. of 3 gives the Log. of 6. The Logarithms of 6 and 2 the logar. of 12, the log. of 6 and 3, the log. of 18, &c.

Thus, The Tabular log. of 3 is = 0,47712127
of 2 is = 0,30103000

of 6 is = 0,77815127

of 12 is = 1,07918127

of 18 is = 1,25527254, &c.

The reason of producing the Logarithms of the powers of a number, by multiplying the Log. of the root by the Exponents

ponents of the power, as likewise of producing of the logarithms of composite numbers by Addition of the logarithms of their composers or factors, is clear from the 14 & 15 of Geometrical progression foregoing, as also from the nature of Logarithms explained in the 177 and 178 pages preceding;

The like operation is to be performed to obtain the Tabular Logarithms of 5, 7, or any other number by their hyperbolick Logarithms, as also the Tabular Logarithms of their powers, or of their Composites, as hath been already performed, for the Tabular Logarithms of 2 and 3, their powers and composites; so that thus much may be deemed sufficient for explaining to any reasonable capacity, the manner, and process how first to calculate the hyperbolick Logarithms, and how from the hyperbolick Logarithms of a few numbers, to make the Tabular logarithms of many, with much facility and dispatch, and that to what exactness or number of places you please, and consequently to calculate a whole Table of logarithms anew, or to examine one already made.

The next Subject that falls under our Consideration as being both proper and very pertinent hereunto, is that of the Logarithmical or Artificial Sines, Tangents and Secants.

The

['T]he Construction of the Tables of Artificial Sines, Tangents and Secants.

T H E Artificial Sines, Tangents and Secants are no other then the Logarithms of the natural Sines, Tan[ge]nts and Secants respectively; being calculated to the na[tu]ral Radius of 1000000000, that so they might be the [mo]re exact. Notwithstanding many natural Tables are [ad]apted to a lesser Radius for more ease in Calculation there[o]f. And from hence it is that the Artificial Radius (*i. e.* [th]e logarithms of the natural Radius) hath always 10 for the [ch]aracteristick, or index thereof, and so the indices of all [th]e Artificial Sines, Tangents and Secants depends on the [nu]mber of places, that the Natural Sines, Tangents and Se[ca]nts consists of, and is either greater or less then 10, accord[in]g as the natural number belonging thereto is either greater [or] lesser then the natural Radius of the same Table: And [fr]om the premises it is clear, that if there were extant a Table [of] logarithms for all numbers from 1 to 1000000000; [th]en might the Tables of Artificial Sines, Tangents and Se[ca]nts be from thence readily Constructed by Transcription [on]ly. Howbeit though no such Table be to be had or expect[ed], yet by a Table of Logarithms for all absolute numbers [fr]om 1 to 1,0000, as in Dr. *Newtons Trigonometria Britannica*, [an]d divers others. The Artificial Sines may (without dif[fic]ulty) be attained to sufficient accuracy by the part propor[tio]nal as follows. And from the Artificial Sines, the Artifici[al] Tangents and Secants is easily made, as shall be hereafter [sh]own in its due place.

Having

Having the Natural Sine of any Arch. To find the Artificial or Logarithmical Sine thereof, by a Table of Logarithms, &c.

1. TAke the first five figures of the Natural Sine of the Arch proposed, to which annex as many cyphers as there are figures remaining, also take another number exceeding the former by an unit with the like number of cyphers annexed to it.

2. Look for the logarithms of the numbers aforesaid prefixing their proper indices.

3. Get the difference of those two numbers, as also the difference of their logarithms, and likewise the excess of the Natural Sine above the lesser absolute number, which is always the remaining figures of the Natural Sine of the Arch proposed.

4. Then say, As the difference of the absolute numbers : Is to the difference of their logarithms : : So is the excess of the Natural Sine above the lesser absolute number : To the excess of the logarithmical sine required above the lesser logarithm. This fourth number or proportional part found, added to the lesser logarithm, gives the logarithm of the Natural sine of the Arch proposed, or Artificial sine required.

First Example.

The Natural Sine of 30 d. is 5000000000, &c. To find the logarithmical or Artificial sine thereof.

Here Note, That in regard the Natural sine of the Arch proposed is 5 with cyphers annexed. Therefore 0,69897000 the logarithm of 5, with (9) the proper index prefixed is the logarithm of 5000000000, or (which is the same) the Artificial or logarithmical sine of 30 d. *viz.* 9,69897000.

Se-

Second Example.

The Natural Sine of 54 deg. 15 min. being 81157,39820. To find the Artificial sine thereof.

The first 5 figures of the Natural Sine, with 5 cyphers annexed is 81157,00000 log. 9,90932598
The numb. greater by 1 is 81158,00000 log. 9,90933134

Diff. of the 2 absol. numbers 100000 dif. log. 536
The excess of the Nat. sine above the lesser absol. numb. 39820

The Proportion is,

As 100000 : 536 : : 39820 : 213

```
  39820
    536
 ------
 238920
119460
199100
 ------
```

The proportional part is 213,43520

Unto	9,90932598	the lesser logarithm
Add	213	the propert. part found.
The sum is	9,90932811	the Artific. sine of 54 d. 15 m.

Here observe that the first number in the proportion is always 1 with cyphers annexed: Therefore the operation for the proportional part is performed by multiplication only.

Also if the Tables of logarithms, for absolute numbers consists of 14 or 15 places; the like method is to be observed for finding the part proportional.

Third

Third Exnmple.

To find the Artificial sine of 60 deg. the natural sine thereof being 8660254038.

The first 5 figures of the Natural sine with 5 cyphers annexed thereto 86602,00000 loga. 9,93752792
The numb. great. by 1 is 86603,00000 loga. 9,93753294

Diff. of the two absol. numb. is 100000 dif. loga. 502
The excess of nat. sine above the less absol. numb. is 54038

The Proportion is,

As 100000 : 502 : : 54038 : 271 :

502

108076
2701900

Unto the proport. part is 271,27076
Add the lesser log. 9,93752792

The sum is 9,93753063 the Artificial sine of 60 d. or logarithm of 8660254038 (the nat. sine of 60 d.)

Having according to this method or by any other that may be judged more facile or expeditious calculated the Artificial sines, for the first two third parts of the quadrant, the other third part may readily and with ease be supplyed with Artificial sines by this proportion, viz. As the sine of an Arch : Is to the sine of 30 d. : : So is the sine of double the said Arch : To the co-sine of the first Arch.

Thus having the Artificial sines for the first 60 d. To find the rest from 60 to 61, 62, 63 d. &c. to 90 deg. it runs thus.

As

As sine of 29 deg. Co-Arithm.	0,31442878
To sine of 30 deg.	9,69897000
So is sine of 58 deg.	9,92842048
To sine of 61 deg.	9,94181926
Again As sine 28 deg. Co-Arithm.	0,32833071
To Sine 30 deg.	9,69897000
So is sine 56 deg.	9,91857421
To sine 62 deg.	19,94593492

The like operation is to be performed for the rest. Or if you begin at 90 deg. To find the remaining third part of the sines, and proceed to 89 d. 58 min. 89 d. 88 d. &c. to 60 the proportion runs thus.

As the sine of 1 d. Co-Arith.	1,75814469
To the Sine of 30 deg.	9,69897000
So is the sine of 2 deg.	8,54281916
To the sine of 89 deg.	19,99993385
Again As the sine of 2 deg. Co-Arithm.	1,45718084
To the sine of 30 deg.	9,69897000
So is the sine of 4 deg.	8,84358451
To the sine of 88 deg.	19,99973535

And so on for the rest, as also for the deg. and minutes.

Or having calculated the last 60 d. of the quadrant, the first 30 deg. may be readily obtained by the following proportion, being only the converse of the former, *viz.*

As the Co-sine of an Arch proposed : Is to the sine of double that Arch : : So is the sine of 30 deg. To the sine of the proposed Arch : And by this proportion you must proceed (in finding the first 30 deg.) from 30 to 29, 28, 27, to 1 deg. 1 min. Thus

As

As the sine of 61 d.	Co-Arith.	0,05818075
To the sine of 58 d.		9,92842048
So is the sine of 30 d.		9,69897000
To the sine of 29 d.		19,68557123

Again, As the sine of 62 d.	Co-Arith.	0,05406508
To the sine of 56 d.		9,91857421
So is sine the of 30 d,		9,69897000
To the sine of 28 d.		19,66160929

Having the Artificial or Logarithmical sine of any Arch: To find the Artificial or Logarithmical Tangent of that Arch.

THE proportion is the same as in page the 169 where the nat. Tangent is found by the Nat. sine, *viz.*

As the Co-sine of any Arch : Is to the sine thereof : : So is Radius : To the Tangent of that Arch: Therefore universally. The Artificiai Co-sine substracted from the sum of Radius and the Artificial sine leaves the Artificial Tangent of that Arch.

Example. To find the Artificial Tangent of 30 d.

As the Co-sine of 30 d.	9,93753063
To the sine of 30 d.	9,69897000
So is Radius	10,00000000
To the Tangent of 30 d.	9,76143937

Th

Thus having made the Artificial Tangents for one half the quadrant, the Artificial Tangents for the other half easily thereby produced by this proportion. As the Tangent : Is to Radius : : So is Radius to the Co-tangent. That is the Artificial Tangent substracted from twice Radius leaves the Artificial Co-tangent.

And hence it is that, the Artificial Tangents and Co-tangents are always the Arithmetical Complements of each other to twice Radius.

Example. For the Tangent of 60 d. the Co-tangent of 30 d.

As the Tangent of 30 d.	9,76143937
To Radius	10,00000000
So is Radius	10,00000000
To the Tangent of 60 d.	10,23856063

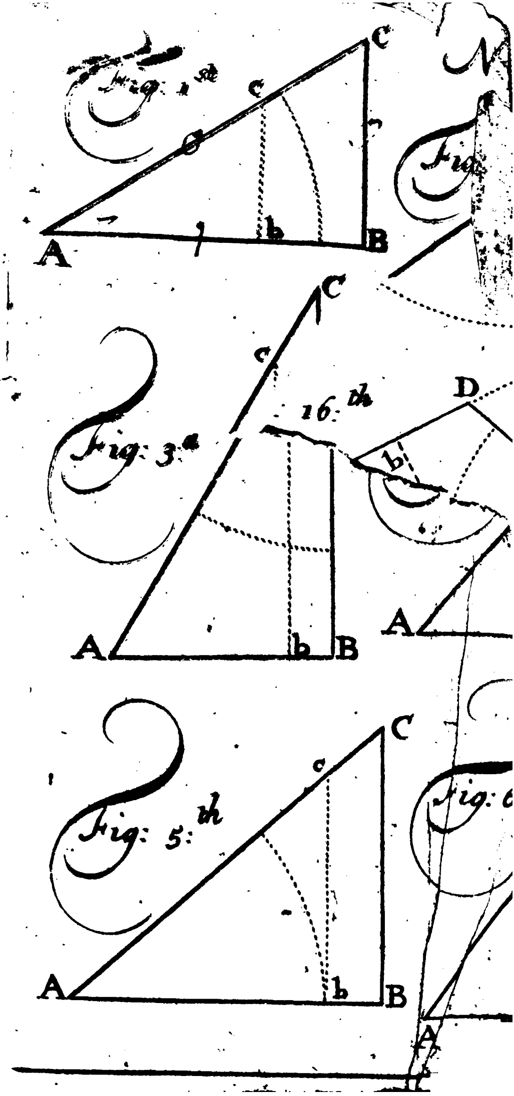
Also by the Artificial sines the Artificial Secants, may be made by this proportion following, which is the same as pag. 169. for finding the natural Secant by the natural sine, *viz.*

As the Co-sine of an Arch : Is to Radius : : So is Radius To the Secant of that Arch : that is the Artificial Co-sine substracted from the doub. of the Artificial Radius, leaves the Artificial Secant of that Arch, and consequently the Secant and Co-Sine of an Arch are always the Complement Arithmetical each to twice Radius.

Example. To find the Secant of 30 d.

As the Co-sine of 30 d.	9,93753063
To Radius	10,00000000
So is Radius	10,00000000
To the Secant of 30 d.	10,06246937

Fo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C
c
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To Radius
So is Radius

Fig: 15:th

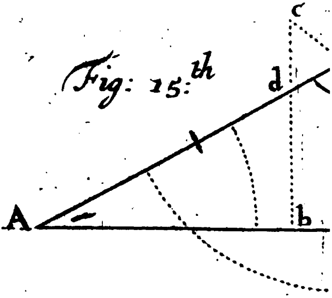


Fig: 16:th

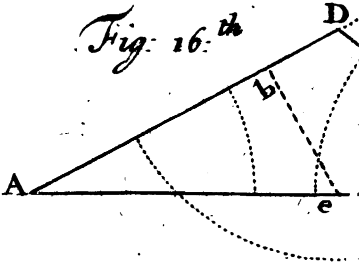
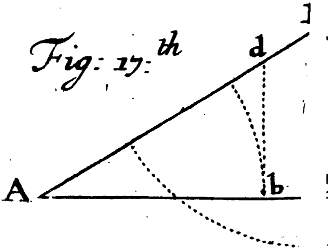


Fig: 17:th



For the Secant of 60 d.

As the Co-fine of 60 d.	9,69897000
To Radius	10,00000000
So is Radius	10,00000000
To the Secant of 60 d.	10,30103000

From hence it is clear, That if to the complement Arithmetical of the Co-fine of any Arch you add Radius, the fum is the Secant of that Arch, and accordingly in fome Tables of Artificial Sines and Tangents instead of the Artificial Secants, are placed the complement Arithmetical of the Sines and Co-fines to Radius, which are no other then the Artificial Secants without their characteristicks or indices, whereby the Secants are eafily fupplyed as aforefaid.

Thus 9,99753068 the Artificial co-fine of 30 d. (or Sine of 60 d.) fubftracted from 10,00000000 the Radius leaves 0,06246937 the complement Arithmetical thereof. Therefore,

Unto 0,06246937 the co-arith. of the Sine of 60 d.
Add Rad. 10,00000000

The fum is 10,06246937 the Secant of 30 d. as before.

Alfo unto 0,30103000 the co-Arith. of the Sine of 30 d.
Add Radius 10,00000000

The fum is 10,30103000 the Secant of 60 d. as before found.

Likewife unto 2,05915815 the Co-Arith. of the Sine of (30 m.
Add Radius 10,00000000

The fum is 12,05915815 the Secant of 89 d. 30 m.

P Note,

Note, That the complement Arithmetical of any Logarithm is most readily had, by subtracting every figure from 9 (beginning at the left hand) and the last towards the right hand from 10.)

But if you are to take the Complement Arithmetical of an Artificial Sine or Tangent greater then Radius. Take it from twice Radius, or (beginning at the left hand) substract the first figure from 19, and each of the rest from 9 only the last from 10 as before, and then in this Case you are to deduct twice Radius, from the sum Resulting by the operation, but in the former but once Radius.

Thus much (considering this Miscellaneous Epitomy) is what seems needful, as being sufficient for the Construction of both the Natural and Artificial Cannon.

As to the Tables themselves it is not thought needful to add any here, or to insist upon their various uses in several parts of the Mathematicks, for the Tables are very plentiful, and their uses largely handled, by divers good Authors such as have been already mentioned.

To

To Calculate TABLES of Randoms in GUNNERY.

THE method hitherto known according to Art and Proportion, published and approved by some Authors of good esteem; is as follows.

First, having made tryal of a piece of Ordinance at any degree of Mounture or Elevation. The Horizontal Ranges are in Proportion to one another. As are the Sines of the double Angle of Elevation: That is having the greatest Random which is at 45 degrees of Elevation, which suppose to be 2000, To find the Range of the shot at 5 degree of Elevation the Proportion is

As Sine of twice 45 d. that is 90 d. or Rad. 10,0000000

To 2000 the greatest Range 3,3010300

So is Sine of twice 5 deg. that is Sine of 10 d. 9,2396702

To 347 the Range at 5 d. Elevation 2,5407002

The like Proportion is to be repeated for the Range, at any other degree of Elevation.

Or, Having the Random at any other degree, as suppose at 20 deg. of Mounture, to be 1286 and from that to Calculate either the greatest Random, or the Randoms for any other degree of Elevation.

For

For the greateſt Random.

As Sine of 40 deg.	9,8080675
To 1286	3,1092401
So is Radius	10,0000000
To the greateſt Random 2000	3,3011726

Or, For the Random at 36 deg.

As Sine of 40 deg. Co-Arith.	0,1919325
To 1286	3,1092410
So is Sine of 72 d.	9,9782063
To 1902 the Random at 36 d.	3,2793798

According to this method (admitting of this Proportion) may a Table of Randoms be Calculated, for any peice of Ordnance to any degree and minute of Elevation by having only one Random at any Elevation: and according to this Proportion those Tables in Mr. *Andersons* Genuine uſe of the Gun were Calculated by Mr. *Street*, but herein Experience is rather to be preferred and chiefly regarded.

FINIS.